Blocking entry in a timing game with asymmetric players*

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Abstract

We examine innovation as a market-entry timing game with complete information and observable actions, allowing for heterogenous players and for multi-peaked and non-monotonic leader payoffs. Assuming that the follower's payoff is non-increasing with the time of the leader's entry, we characterize all pure-strategy subgame perfect equilibria for the two-player asymmetric model, showing that there are at most two equilibria. Moreover, firm heterogeneity allows for equilibria with different types of characteristics than previously examined in the literature. For example, anticipating that it will be preempted by its rival, a firm may opt to enter even earlier, effectively *blocking entry*. Our general framework also allows us to analyze comparative statics relating to the timing of entry. In a tale of caution for policy makers, unlike with symmetric firms, our results indicate that with heterogenous firms the timing of entry (and the technology adopted) could respond ambiguously to changes in payoffs.

Key words: timing games, entry, leader, follower, process innovation, product innovation.

JEL classifications: C72, L13, O31, O33.

1 Introduction

When should a firm innovate or launch a new product? Sometimes it is better to be first into a market. Reinganum (1981a,b) shows that when firms can observe their

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rival's actions (in an open-loop equilibrium), the leader can receive a higher payoff than the follower. Moreover, the second entrant might end up entering much later than the leader, even though the duopolists are ex ante identical. The conventional business-press wisom of a first-mover advantage does not universally hold; that is, a market leader need not always be better off. Fudenberg and Tirole (1985) show that when rivals' actions are observable (in a closed-loop equilibrium) the incentive to preempt can dissipate all potential gains from entering first, equalizing rents to both firms in the process. Moreover, in many situations it can be advantageous to enter the market second, rather than first, as the market pioneer might need to incur set-up or R&D costs on which its rival can free-ride.

Several themes run through this existing literature. Firstly, the presence of a rival(s) complicates a firm's entry decision, given the potential strategic interaction. Secondly, this interaction can induce inefficient entry (Fudenberg and Tirole, 1985). Drawing on these themes we study a novel duopoly model of innovation that allows for: (i) heterogeneous firms; and (ii) the possibility that alternative technologies become available (at a later date) if a firm delays entry. We provide a general solution method and completely characterize all pure-strategy subgame perfect equilibria. In this framework we show that new inefficiencies can arise; not only is it the case that firms can choose to enter the market at the wrong *time*, they can also choose to enter the market with the wrong *technology*.

The basic features of our model are as follows. Two firms can make an irreversible and one-off decision to enter a market. Time is continuous and all previous actions (entry or not) are observable; consistent with this, we focus on closed-loop equilibria. In Fudenberg and Tirole (1985) and others, the entrants are ex ante identical and have access to the same potential innovation. But usually firms are not all the same. Drawing inspiration from Katz and Shapiro (1987), we study two heterogenous firms that can have different payoffs from entering at a given time.[1] This assumption of heterogeneity is widely applicable. Firms might differ in their ability to exploit market opportunities, for instance. The expected payoffs could differ between two rivals considering launching a new phone handset or tablet (with equivalent functionality) given their existing reputation or network. It could also depend on the other tie-in products they have to offer. The same can be said for a process (cost-saving) innovation – its payoff depends on access to markets, how the new technology meshes with a firm's existing practices, and

[1]Katz and Shapiro (1987) analyze an innovation game with heterogenous firms when there is licensing (by the leader) and imitation (by the follower). They find that industry leaders (who are more efficient) need not be the firm that innovates, as it may prefer to free ride on the public good provided by its rival. Riordan (1992) uses a similar framework to examine the impact of regulation of technological adoption. Also see Galasso and Tombak (2014), who adapt Katz and Shapiro (1987) to study the take-up of green technologies that have both a private and public good benefit by asymmetric firms.

so forth.

As noted above, firms often also have to choose which technology to implement when they enter the market. Returning to the smartphone example, Samsung made a choice to switch its cell phone operating system from its own in-house system to an Android platform. Sony also made an equivalent choice. Despite its closed system, in many ways Apple faces a similar tradeoff when it considers the introduction of iOS for its devices. Implicit in this is that not all technologies are available immediately; rather, some technologies are only available (or worth considering) later. To capture this, unlike in Katz and Shapiro (1987), we allow the leader's payoff to be multi-peaked with respect to its entry time. This payoff structure, generated by the choice between multiple technologies, combined with the asymmetric payoffs between players, creates a new strategic entry environment not previously analyzed.

While we place effectively no restrictions on the leader's payoff function other than continuity, in a similar way to Hoppe and Lehmann-Grube (2005) and Argenziano and Schmidt-Dengler (2012, 2013, 2014), we assume that the payoff of the follower is non-increasing with leader's time of entry. This could be the case, for instance, when later entry by the leader (conditional on it still being first) affords it to enter the market with a better (less costly) production technology or product, or possibly both, which in turn exerts greater competitive pressure on the second entrant.

Adapting the solution technique of Smirnov and Wait (2015) to asymmetric firms, we characterize all pure-strategy subgame perfect equilibria. In any entry game in our setting we find that there can be zero, one or two pure-strategy equilibria. Just like in Fudenberg and Tirole (1985) and Katz and Shapiro (1987), we show that there can be a preemption equilibrium, in which a firm enters before the *stand-alone* entry time.[2] It is also possible that there is a second-mover advantage equilibrium.[3] In this type of equilibrium, while both firms would prefer to enjoy the spoils of being second, one of the two needs to self sacrifice and enter first. Firm heterogeneity, however, allows for another, even more nuanced, possibility. As it turns out, with asymmetric payoffs it is feasible that one of the firms wishes to be a leader while, at the same time, the other would prefer to be a follower. In this way, firm asymmetries can allow leader and follower advantages to co-exist in equilibrium.

Given its generality, our model incorporates equilibria highlighted in the pre-

[2]This terminology follows Katz and Shapiro (1987), in which the *stand-alone* entry time is time of entry a firm would choose if it faces no threat of entry by a rival.

[3]Theoretically, second-mover advantages with observable actions have been studied by Dutta et al. (1995), Hoppe (2000), Hoppe and Lehmann-Grube (2001), Hoppe and Lehmann-Grube (2005) and Smirnov and Wait (2007, 2015). Also see the empirical findings of Tellis and Golder (1996), who show that early imitators often outperform market pioneers.

vious literature, such as the preemption and second-mover advantage equilibria. However, heterogeneity of firm payoffs allows for equilibria with novel yet empirically relevant characteristics. In our analysis we show that it is not only possible that the timing of entry is inefficient, but that the leader enters with less efficient technology. This arises under plausible economic scenarios when, for example, a firm anticipating that it will be preempted by its rival, opts to enter even earlier (with the less efficient technology). This *blocking entry* equilibrium is only possible with heterogeneous firms, and could result in multiple inefficiencies, with respect to the type of technology adopted, the timing of entry and even which of the firms enters as the market leader. Our model is also a tale of caution for policy makers who wish to influence the timing (and technology chosen) in a particular market. Unlike the symmetric case where the direction of change is clear, with heterogenous firms we show that when the leader and follower entry payoffs change (perhaps due to a subsidy or tax break), the effect on the timing of entry is ambiguous.

Our *blocking entry* outcome is potentially empirically relevant. It also has links to the blocking/accommodation literature, as summarized in Tirole (1988), in which a firm (the incumbent) invests in the first period (in R&D, development of patents, production capacity, investment to reduce costs, advertising, and so on) in anticipation of its effect on the ex post competition. In this setup, an incumbent might wish to invest in order to block entry by a potential rival, staying as a monopolist in the market. Alternatively, the incumbent might be better not blocking but accommodating entry by its rival, but it will still strategically invest with an eye on its returns in the duopoly market.[4]

Adapting this theoretical framework, Gil et al. (2015) empirically investigate preemption in the US drive-in cinema market, and find a non-monotonic relationship between market size and preemption; whilst early entry will have little impact on the final market structure in either very small or very large markets, it is the mid-sized markets in which there is the greatest incentive to preempt a rival.[5] Similarly, Schmidt-Dengler (2006) apply a timing-game framework to study preemption and business stealing in relation to the adoption of MRI technology by US hospitals. As in these two empirical studies, the timing of entry in our model depends on the strategic interaction between the firms and, in particular, the threat of entry by a rival can induce early entry by a firm. This sorts of strategic issues are also discussed in the business press. The fierce rivalry between Apple and Samsung in the smartphone market, for instance, manifests itself in

[4]The subtly in these models comes from the interaction between the incumbents investment and the nature of ex post competition. See Tirole (1988) and Fudenberg and Tirole (1984), for example.

[5]Also see Ellison and Ellison (2011) for an application to blocking and accommodating investment by pharmaceutical companies relating to drugs coming off patents.

no small way in the launch dates for new versions of the iPhone or the Galaxy.[6] Our *blocking entry* equilibrium suggests that the threat of preemption can induce a firm to enter so much earlier than its preferred time that it requires launching an inferior product or technology, or one that is really not ready for market.

This paper draws on an extensive literature on innovation timing games.[7] Our analysis of an irreversible investment decision with complete information and observable actions (closed-loop equilibria) follows Fudenberg and Tirole (1985), Dutta et al. (1995), Hoppe and Lehmann-Grube (2005) and Smirnov and Wait (2015). This framework has been used to study a range of applications. For example, Argenziano and Schmidt-Dengler (2012, 2013, 2014) adopt a variant of Fudenberg and Tirole (1985) to examine the order of market entry, clustering and delay. They show that with many potential entrants the most efficient firm need not be the first to enter the market and that delays are non-monotonic with the number of firms. In addition, they suggest a new justification for clustering of entry. Others have studied similar issues. Anderson et al. (2017) studies delays and rushes into a market in a stopping game with a continuum of players.[8]

While we assume that previous actions of a rival are observable, an alternative approach to study innovation is to assume players' actions are unobservable as in Reinganum (1981a,b), where unobservable actions are equivalent to each firm being able to pre-commit. Reinganum shows that in the open-loop equilibria there will be diffusion in the sense that firms adopt the technology at different dates, even though all firms are ex ante identical. Similarly, Park and Smith (2005) develop an innovation game with unobservable actions that permits any firm (in terms of the order of entry) to receive the highest payoff. This allows for a war-of-attrition, with higher payoffs for late movers, a pre-emption game with higher payoffs for early movers, and a combination of both. An important point of comparison is that in our model firms use feedback rules to determine their strategy at any particular point in time; this means that they are unable to commit to their strategy at the beginning of the game.

Information also plays a key role in the players' entry strategies. Bloch et al. (2015) show that when two potential rivals are uncertain about their entry costs, competition leads to inefficient entry that is too early. Other authors consider inefficiencies in innovation when there is asymmetric information. For example, Bobtcheff and Mariotti (2012), Hendricks (1992) and Hopenhayn and Squintani (2011) assume that a firm's capability to innovate is private information. In these models, delay allows a firm to get better information about the potential innovation

[6]See, for example, 'Phone tag; Apple v Samsung' in *The Economist*, September 16 2017.

[7]See Hoppe (2002) or Van Long (2010, Chapter 5) for a survey of the literature. Further, Fudenberg and Tirole (1991) consider innovation when the firms make one irreversible decision (to enter) in a simple timing-game framework (see Sections 4.5 and 4.12).

[8]See also Riordan (1992) and Alipranti et al. (2011, 2015).

(its costs, value, and so on), but waiting runs the risk that a rival will innovate first, capturing the lion's share of the returns.

2 The model

Assume two firms ($i = 1, 2$) are in a continuous-time stopping game starting at $t = 0$ until some terminating time $T \in (0, \infty]$. Firm i's one-off decision to stop (that is, 'enter' the market) at $t_i \geq 0$ is irreversible and observable immediately by the other firm. The game ends when one of two firms has stopped/entered the market. The payoff to each firm depends on the stopping time. If the game ends with player i stopping at time t_i, the payoffs of the leader and the follower are $L_i(t_i)$ and $F_j(t_i)$, respectively, where $i, j = 1, 2$ and $i \neq j$.

We make the following assumptions.

Assumption 1. *Time is continuous in that it is 'discrete but with a grid that is infinitely fine'.*

Assumption 2. *Firms always choose to stop earlier rather than later in payoff-equivalent situations.*

Assumption 3. *If more than one firm chooses to stop (enter) at exactly the same time, one of these firms is selected to stop (each with an ex ante probability of $\frac{1}{2}$).*

Entry models in the literature adopt equivalent assumptions. Assumption 1 invokes Simon and Stinchcombe (1989) who show that under certain conditions a continuous-time strategy profile is the limit of a discrete-time game with increasingly fine time grids. It also replicates A1 of Hoppe and Lehmann-Grube (2005).[9] Assumption 2, which is similar to A3 in Hoppe and Lehmann-Grube (2005), allows us to focus on just one (payoff-equivalent) equilibrium in the case of indifference between early and late entry.[10] This simplifies our analysis so as to focus on the timing of entry rather than on issues of equilibrium selection.

Assumption 3 – part of A3 in Hoppe and Lehmann-Grube (2005) and Assumption 5 in Dutta et al. (1995) – avoids potential coordination failures involving simultaneous entry. Given its importance, the intuition underlying this assumption warrants further discussion. In some situations, as a practical matter, if two firms try to enter the market at the same time there might be some capacity constraint or institutional requirement that prevents joint entry – consequently, one firm becomes the leader and the other firm is relegated to the role of second

[9] See Hoppe and Lehmann-Grube (2005), footnote 4 for a further discussion.

[10] Hoppe and Lehmann-Grube (2005) assume that if the follower is indifferent between two alternative entry times, it chooses the earliest time of entry.

entrant. For instance, in a particular market there could be a bureaucratic rule that requires the leadership role be allocated to the firm that has the first email registered in a designated inbox. Even if both firms simultaneously send their messages, only one email can arrive first. As a consequence, with simultaneous moves, each firm has some probability of being the leader. Equivalent intuition applies to any (bureaucratic) tie-breaking rule that determines the winner in what seems to be a dead heat. Dutta et al. (1995) present a similar rationale for this assumption, suggesting there could be small random delays between when a decision is made and when a new technology is adopted, meaning that there is a positive probability that either firm is first in the event of joint adoption. Here, Assumption 3 gives both firms an equal chance of being first when there is simultaneous entry.

The following two assumptions ensure that the leader stops in finite time.[11] The first element of this is that leaders' payoff functions reach their respective global maxima at a finite point in time; this means that both firms will not delay entry indefinitely. Dutta et al. (1995) (Assumption 3), Fudenberg and Tirole (1985) (Assumption 2(ii)) and Smirnov and Wait (2015) (Assumption 4) all make equivalent assumptions. Secondly, we assume that entering provides a higher payoff than each firm's respective outside option of zero, thus ensuring that our analysis is not unnecessarily complicated by having to consider whether one or both firms never enter the market. Again, this mirrors assumptions made previously in the literature, such as Assumption 2(ii) in Fudenberg and Tirole (1985), Assumption 4 in Dutta et al. (1995) and Assumption 5 in Smirnov and Wait (2015).

Assumption 4. *There exists a finite $\widehat{T}_i < T$, which is the earliest time at which $L_i(t)$ attains its global maximum. Specifically, $L_i(\widehat{T}_i) > L(\tau)\ \forall\ \tau < \widehat{T}_i$, and $L_i(\widehat{T}_i) \geq L(\tau)\ \forall\ \tau \geq \widehat{T}_i$ where $i = 1, 2$.*

Assumption 5. *Each firm's outside (non-entry) payoff is normalized to 0, and $L_i(t) \geq 0$ and $F_i(t) \geq 0\ \ i = 1, 2$.*

Finally, we assume that the advantage of being second is non-increasing with the leader's time of entry. This follows Hoppe and Lehmann-Grube (2005) who employ a similar assumption. In addition, this assumption incorporates the scenario studied in Argenziano and Schmidt-Dengler (2012, 2013, 2014), in which the payoff of the follower is constant with respect to the leader's entry time.[12]

There are several possible explanations for why the follower's payoff would be non-increasing in the leader's entry time. If either cost fall or there is an

[11]When there is no ambiguity, we refer to payoffs as a function of t rather than t_1.

[12]It is worth noting that here we assume one potential innovation implemented by the market leader. In Argenziano and Schmidt-Dengler (2014), on the other hand, they model explicitly both firms entering the market. In equilibrium in their model entry by the follower always occurs at some later fixed date, resulting in a constant payoff for the follower.

improvement of the quality of the product with time, later entry by the leader could place the follower at a relative disadvantage; any delay in the initial entry time could help make the leader a stronger competitor, other things equal, hurting the firm that enters the market second. It is worth noting that this assumption is not crucial; rather, our key results hold in a more general environment. Assuming that the follower's payoff is non-increasing in the leader's time of entry, however, helps highlight the key economics insights of the model. We discuss this issue further in Section 5.

To aid in exposition, we restrict our analysis to continuous leader and follow payoff functions. A detailed analysis solving entry games with discontinuous (but symmetric) payoffs can be found in Smirnov and Wait (2015).

This discussion is summarized in following assumption.

Assumption 6. *$L_i(t)$ is continuous, while $F_i(t)$ is continuous and non-increasing for $i = 1, 2$.*

In summary, the first five assumptions are standard in the market-entry timing game literature with complete information and observable actions; see for example Smirnov and Wait (2015). Our last assumptions is similar to Argenziano and Schmidt-Dengler (2012, 2013, 2014) and Hoppe and Lehmann-Grube (2005), however we allow for more generality in the structure of payoffs.

To conclude this subsection, we outline two useful definitions. Firstly, provided $L_i(t)$ and $F_i(t)$ cross at least once, following Katz and Shapiro (1987), we define $\widetilde{T}_i$ to be the earliest time the payoff functions intersect.

Definition 1. *If $L_i(t)$ and $F_i(t)$ intersect, $\widetilde{T}_i \leq T$ is the earliest time at which $L_i(t) = F_i(t)$.*

Secondly, we will use the following definition.

Definition 2. *Define $\bar{T}_2 \leq \widetilde{T}_2$ to be the earliest time at which $L_2(t)$ attains its maximum for $t \in [0, \widetilde{T}_2]$.*

2.1 Equilibrium concept

Following Fudenberg and Tirole (1985), we use subgame perfection. A history h_t is defined as the knowledge of whether or not firm $i = 1, 2$ previously stopped at any time $\tau < t$, and if so when. A strategy of firm i, denoted by $\sigma_i(h_t)$, indicates at each history h_t whether firm i stops at t ($\sigma_i(h_t) = 1$) or does not stop at t ($\sigma_i(h_t) = 0$). A strategy pair (σ_1, σ_2) maps every history to an outcome, which is the minimum of stopping times t_1 and t_2. As usual, a strategy profile (σ_1^*, σ_2^*) constitutes a subgame perfect equilibrium (SPE) if the strategies are sequentially rational after every history. Note here that with this representation we only need to specify the strategies when there has been no entry in the history of the game, because we assume that once one firm has entered, the game ends (Katz and Shapiro, 1987). This allows us, for ease of exposition, to refer to each firm's entry strategy as a function of time only, $\sigma_i(t)$.

3 Characterization of equilibria

In this section we first describe equilibria in the case of symmetric firms, before exploring market entry when the firms potentially have different payoffs (Section 3.2).

3.1 Symmetric firms

To outline a benchmark for the analysis that follows, first assume that both firms are the same in terms of their potential payoffs. The proposition below describes the method for determining the entry time of the leader in the symmetric case.

Proposition 1. *[Smirnov and Wait (2015)] The equilibrium of the symmetric model is always unique. The first firm's stopping time t^* is given by*

$$t^* = \min \arg \max_t \min[L(t), F(t)]. \tag{1}$$

As outlined in Smirnov and Wait (2015), this algorithm takes the minimum of the payoff functions for the leader and the follower, respectively. If the leader's payoff at the start of the game exceeds (or is equal to) the follower's payoff, given the follower's payoff is non-increasing, immediate entry ($t^* = 0$) maximizes the minimum of the two payoff functions (or is the earliest time to do so). In this case there is a first-mover advantage (or rents are equalized if $L(0) = F(0)$). Consider now the case when at the start of the game the follower's payoff exceeds that of the leader. Given that the follower's payoff is non-increasing, the first intersection between the two payoff functions ($\widetilde{T}$) is the only intersection that is economically relevant. If the leader's payoff is at its historical maximum at $\widetilde{T}$, entry occurs at this time (equalizing rents). This situation is illustrated in Figure 1(a). The bold line traces out the minimum of the leader and follower payoff functions. On the other hand, consider the situation when the leader's payoff at $\widetilde{T}$ is not at its historical maximum; see Figure 1(b). In this case there are two second-mover advantage equilibria in which one of the firms enters at t^*, while the other enjoys a higher payoff as the follower. Finally, when there is no intersection between $L(t)$ and $F(t)$, and $F(t)$ always exceeds $L(t)$, the leader enters at the time that $L(t)$ attains its global maximum ($\widehat{T}$). Again, there are two pure-strategy equilibria with a second-mover advantage, which entail either one of the firms acting as the market leader.

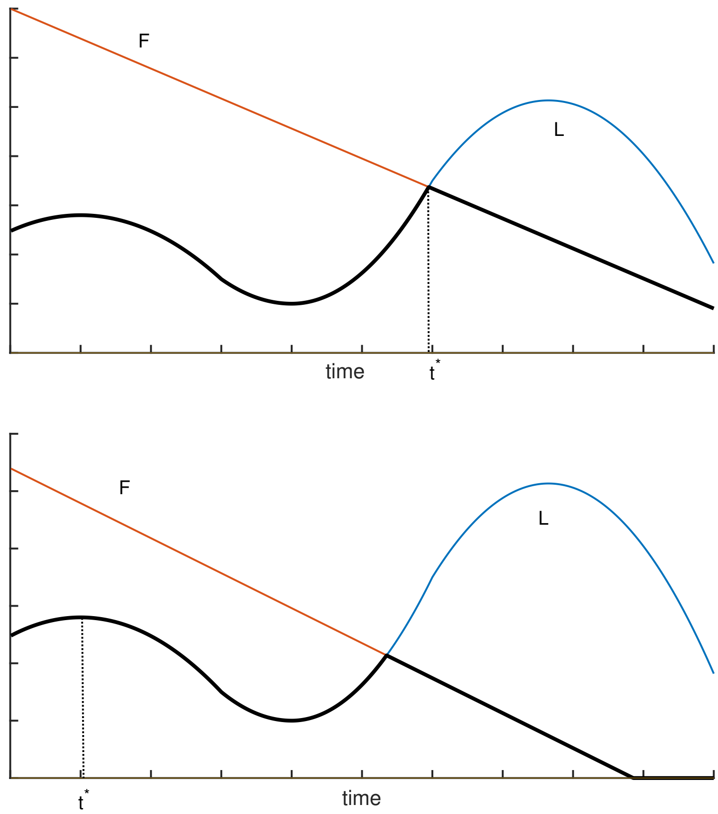

Figure 1: Preemption and second-mover advantage with symmetric firms

3.2 Asymmetric firms

As noted previously, firms are more often than not different from one another. In this section we develop a method of determining the leader's entry time in all pure-strategy SPE, allowing for asymmetric payoff functions. Firstly, to find the pure-strategy SPE we note that any equilibrium with player i entering at time t_i must satisfy two necessary conditions:

Condition 1. *No preemption by the leader i (NPL):* $L_i(t_i) > L_i(\tau), \ \forall \ \tau \in (0, t_i)$.

Condition 2. *No preemption by the follower j (NPF):* $F_j(t_i) > L_j(\tau), \ \forall \ \tau \in (0, t_i)$ *and* $F_j(t_i) \geq L_j(t_i)$.

If the *NPL* does not hold, the leader (player i) will deviate by entering earlier. Similarly, the *NPF* must hold in any SPE, otherwise the follower (player j) has an incentive to preempt and enter slightly earlier than the leader, as in Fudenberg and Tirole (1985).[13] Even if these conditions hold, they do not in of themselves guarantee that a specific entry time is part of an SPE, because both only compare payoffs at a particular time relative to their historic values. These conditions, by definition, do not make any comparisons with future potential payoffs. Of course, such a consideration is necessary when determining any SPE.

To solve for the leader's entry time, let us eliminate all points that do not satisfy either of these conditions (the *NPL* and the *NPF*) by constructing sets $A_1(t', t'')$ and $A_2(t', t'')$. For each firm $i \in \{1, 2\}$, $j \neq i$ and $t'' > t' \geq 0$, define the following set:

$$A_i(t', t'') = \{ \ t \in (t', t''] \ | \ L_i(t) > L_i(\tau) \ \& \ F_j(t) > L_j(\tau) \ \forall \ \tau \in (t', t) \ \& \ F_j(t) \geq L_j(t)\}. \tag{2}$$

By definition, a point belongs to set $A_i(t', t'')$ if it satisfies both *NPL* and *NPF*. By way of comparison, to solve the symmetric-player entry game Smirnov and Wait (2015) construct one set that is applicable to both firms. Here, asymmetry requires the construction of a set $A_i(.)$ for each firm and for any truncated game played on interval $[t', t'']$.

For each firm $i \in \{1, 2\}$ define the following time

$$t_i^* = \begin{cases} \arg\max\limits_t A_i(0, T) & \text{when} \quad A_i(0, T) \neq \emptyset, \\ 0 & \text{when} \quad A_i(0, T) = \emptyset. \end{cases} \tag{3}$$

In addition, assume without loss of generality that $t_1^* \geq t_2^*$. Moreover, for the truncated game played on $[0, t_2^*]$ define the following time

$$t_1^{**} = \begin{cases} \arg\max A_1(0, t_2^*) & \text{when} \quad A_1(0, t_2^*) \neq \emptyset, \\ 0 & \text{when} \quad A_1(0, t_2^*) = \emptyset. \end{cases} \tag{4}$$

[13] Argenziano and Schmidt-Dengler (2014) adopt similar conditions, which they refer to as the *Leader Preemption Constraint* and the *Follower Preemption Constraint*.

Now we are in the position to characterize all SPE of the game with asymmetric payoff functions, as summarized in the following proposition.

Proposition 2. *Consider the SPE of the two-player asymmetric timing game. If*

1. $A_1(t_1^*, T) = \emptyset$ *and* $L_1(t_1^*) > F_1(t_1^*)$*, the SPE involves firm 1 entering at* $t = t_1^*$*;*
2. $A_1(t_1^*, T) = \emptyset$ *and* $L_1(t_1^*) \leq F_1(t_1^*)$*, there are two SPE, one with firm 1 entering at* $t = t_1^*$ *and the other with firm 2 entering at* $t = t_2^*$*;*
3. $A_1(t_1^*, T) \neq \emptyset$
   (a) $t_2^* < \bar{T}_2$ *and* $A_2(t_2^*, t_1^*) = \emptyset$*, there is no SPE;*
   (b) $t_2^* = \bar{T}_2$ *and* $A_2(t_2^*, t_1^*) = \emptyset$*, there is a unique SPE with firm 2 entering at* $t = t_2^*$*;*
   (c) $A_2(t_2^*, t_1^*) \neq \emptyset$*, there is a unique SPE involving firm 1 entering at* $t = t_1^{**}$*.*

Proof: See Appendix A.

To help in outlining the intuition underlying Proposition 2, consider the following series of corollaries, first starting with the simplest scenario when the leader payoff functions are monotonically increasing until their global maxima $\widehat{T}_i$, for $i = 1, 2$, respectively.

Corollary 1. *Assume that for* $i = 1, 2$*:* $L_i(t)$ *and* $F_i(t)$ *cross at least once;* $L_i(t)$ *is a monotonically increasing function for* $t \in [0, \widehat{T}_i]$*; and* $\widehat{T}_1 \geq \widetilde{T}_1$*. In the SPE of the two-player asymmetric timing game:*

1. *if* $t_1^* > t_2^*$ *the SPE is unique and involves firm 1 entering at* $t = t_1^*$*;*
2. *if* $t_1^* = t_2^*$ *there are two SPE that involve either firm entering at* $t = t_1^*$*.*

When: (i) both leader curves intersect with their corresponding follower curves; (ii) the leader functions are monotonically increasing functions until they reach their global maxima; and (iii) the maxima occur after the first intersections, there are three possible equilibrium outcomes. As explained below, all three involve preemption by firm 1. The first possibility is illustrated in Figure 2.[14] In this case the global maxima for both leader payoff functions are at an entry time after t_2^*. Given the incentive to preempt, entry will occur at a time before either of these maxima. Rather, firm 1 enters at $t_1^* = \widetilde{T}_2$ (where L_2 crosses F_2) so as to just preempt entry by its rival.[15]

[14] Both in this example and in most of the examples that follow we make the simplification that $F_1 = F_2$ for illustrative purposes.

[15] Note that by construction $\widetilde{T}_1 < \widetilde{T}_2$.

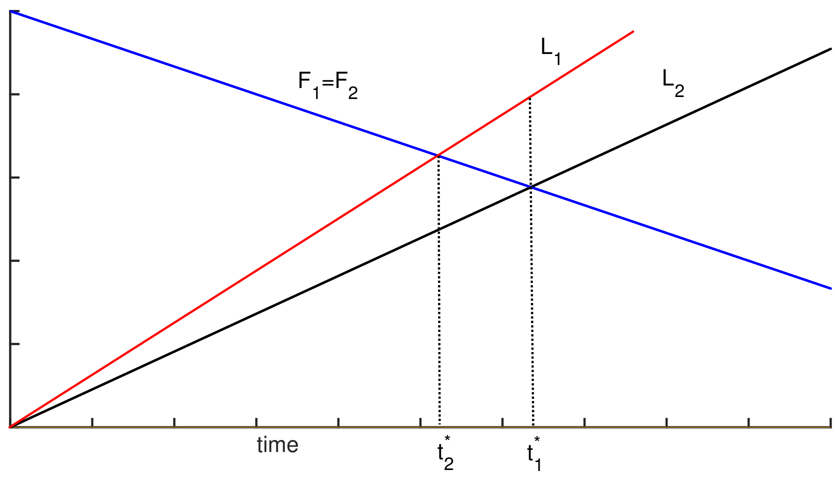

Figure 2: Both leader's payoffs are monotonically increasing functions

Now consider the case when $\widetilde{T}_1 < \widehat{T}_1 < \widetilde{T}_2$. Firm 1 will enter at the time that maximizes its leader payoff, $\widehat{T}_1$. Again there is a unique time of entry – t_1^* – but in this case firm 1 has a first-mover advantage whereas firm 2 prefers to be the second mover.

Finally, there is also the possibility that the first intersection between either leader payoff functions and the follower payoff curve occurs at the same time – that is $\widetilde{T}_1 = \widetilde{T}_2$. In this case both firms are in a preemption game. There are two equilibria with either firm entering at $t = \widetilde{T}_1 = \widetilde{T}_2$. Notably, in each of these three scenarios, there is either one equilibrium (with firm 1 entering) or two equilibria (with the same entry time).

Next, let us consider an intermediate scenario where both leader's payoff curves are concave. While similar to the scenario considered in Katz and Shapiro (1987), our Assumption 5 guarantees that one of the firms always (eventually) enters the market.

Corollary 2. *Consider the SPE of the two-player asymmetric timing game when both $L_1(t)$ and $L_2(t)$ are concave (hump-shaped) functions. If*

1. $A_1(t_1^*, T) = \emptyset$ *and* $L_1(t_1^*) > F_1(t_1^*)$, *there is a unique SPE with firm 1 entering at* $t = t_1^*$.

2. $A_1(t_1^*, T) = \emptyset$ *and* $L_1(t_1^*) \leq F_1(t_1^*)$, *there are two SPE, one with firm 1 entering at* $t = t_1^*$ *and the other in which firm 2 enters at* $t = t_2^*$.

3. $A_1(t_1^*, T) \neq \emptyset$

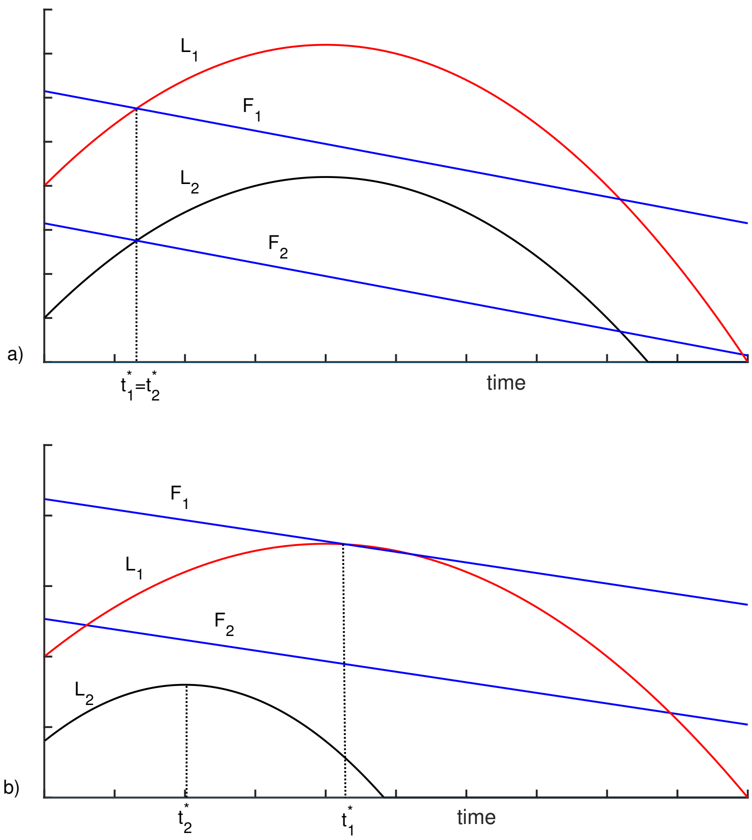

Figure 3: Example of Corollary 2(2) when both leader's payoffs are concave functions, $L_1(t_1^*) \leq F_1(t_1^*)$ and $A_1(t_1^*, T) = \emptyset$: entry occurs at either t_1^* or t_2^*

> *(a) and* $t_2^* < \widehat{T}_2$, *there is no pure-strategy SPE.*
>
> *(b) and* $t_2^* = \widehat{T}_2$, *there is a unique pure-strategy SPE, in which firm 2 enters at* $t = t_2^*$.

With the concave leader payoff functions described in Corollary 2, the SPE need not just involve a preemption equilibrium, which was the case in Corollary 1. Note, first, Corollary 2(1) includes the case covered in Corollary 1(1). This is the case, presented in Figure 3, when both firms have an incentive to preempt, however firm 1 has an advantage in terms of payoffs $(L_1(t_1^*) > F_1(t_1^*))$. Corollary 2(1) also includes the 'mixed' case scenario in which firm 1 prefers to be a leader, entering at $t_1^* = \widehat{T}_1$ (as $L_1(t_1^*) > F_1(t_1^*)$ and $A_1(t_1^*, T) = \emptyset$, meaning that there is no advantage waiting for a higher payoff later), whereas firm 2 prefers to be a follower at t_1^* rather than entering as a leader at t_2^*, because $L_2(t_2^*) < F_2(t_1^*)$.

The strategies firms adopt in this SPE are:

$$\sigma_1(t) = \begin{cases} 1 & \text{if} \quad [A_1(t,T) = \emptyset \ \& \ L_2(t) \leq F_2(t)] \ \& \ [L_1(t) > F_1(t) \ \& \ L_2(t) > F_2(t)], \\ 0 & \text{otherwise;} \end{cases}$$

$$\sigma_2(t) = \begin{cases} 1 & \text{if} \quad L_2(t) > F_2(t), \\ 0 & \text{otherwise.} \end{cases}$$

These strategies require that firm 1 enters in two distinct situations. Firstly, firm 1 opt to be the leader when it has no further incentive to wait in the hope of a higher return later ($A_1(t_1^*, T) = \emptyset$) and firm 2 prefers to be a follower, as $L_2(t) \leq F_2(t)$. The second situation resembles the classic preemption game outlined in Fudenberg and Tirole (1985), as both firms prefer to be a leader rather than a follower when both $L_1(t) > F_1(t) \ \& \ L_2(t) > F_2(t)$. On the other hand, considering the strategy of firm 2, it will only enter the market at t if doing so dominates waiting. This holds when $L_2(t) > F_2(t)$, remembering that $F_2(t)$ is a non-increasing function.

Corollary 2(2) corresponds to two distinct scenarios. Firstly, as in Corollary 1(2), if $A_1(t_1^*, T) = \emptyset$, $t_1^* = t_2^*$ and $L_i(t_1^*) = F_i(t_1^*)$ there are two preemption equilibria, with either firm acting as the leader, $t_1^* = t_2^*$. This is illustrated in the top panel of Figure 3. In one of the equilibria firm 1 enters at t_1^*, while firm 2 waits. Alternatively, firm 1 plays the role of the follower, and firm 2 enters as the market pioneer. In either of the equilibria entry occurs at the same time.

The second scenario covered by in Corollary 2(2) is illustrated in the example shown in the bottom panel of Figure 3. Note that $L_1(t_1^*) \leq F_1(t_1^*)$ and $L_2(t_2^*) < F_2(t_2^*)$. In this case there are two SPE, each with a second-mover advantage. In each of these equilibria either firm enters when they attain their highest leader payoffs $\widehat{T}_i$ (and the other firm always waits, unless entering strictly dominates waiting).

Explicitly, for SPE in Corollary 2(2) where firm i is the leader and firm j is the follower, the firms' strategies in each of the SPE are:

$$\sigma_i(t) = \begin{cases} 1 & \text{if} \quad [A_i(t,T) = \emptyset \ \& \ L_j(t) \leq F_j(t)] \ \& \ [L_1(t) > F_1(t) \ \& \ L_2(t) > F_2(t)], \\ 0 & \text{otherwise;} \end{cases}$$

$$\sigma_j(t) = \begin{cases} 1 & \text{if} \quad L_j(t) > F_j(t), \\ 0 & \text{otherwise.} \end{cases}$$

These strategies generalize the outlined for Corollary 2(1) above. Consider the equilibrium when firm 2 enters, so it plays the role of firm i. Its strategy involves it waiting until its leader payoff is maximized, and then entering at this time. If, somehow, a preemption subgame is reached in which $[L_1(t) > F_1(t)$ and $L_2(t) > F_2(t)$, entering dominates waiting, so firm 2 would enter. Conversely,

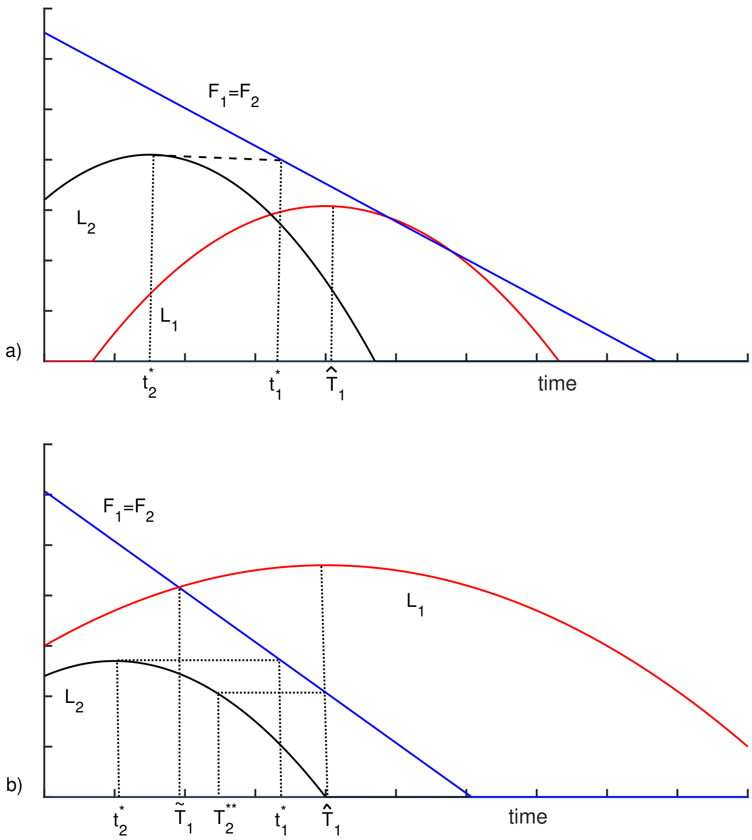

Figure 4: Example of SPE in Corollary 2(3b), when $A_1(t_1^*, T) \neq \emptyset$ and $t_2^* = \widehat{T}_2$: firm 2 is the market leader, entering at t_2^*

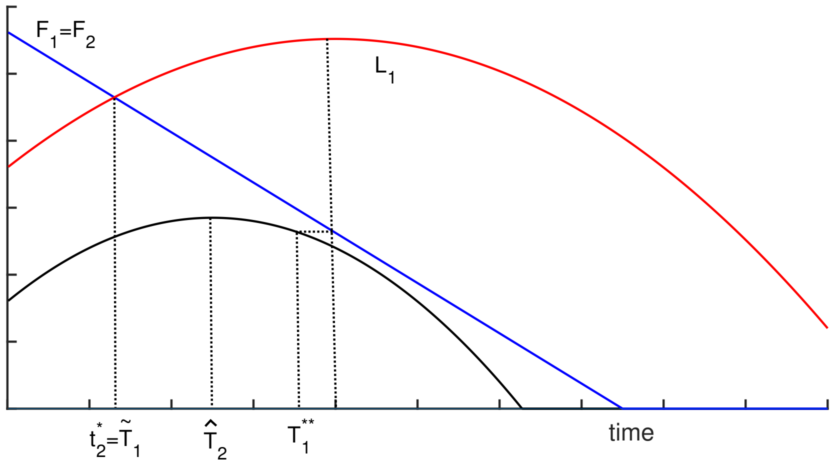

Figure 5: Example of no pure strategy SPE in Corollary 2(3a): $A_1(t_1^*, T) \neq \emptyset$ and $t_2^* < \widehat{T}_2$

firm 1 plays the role of follower in this cae. It will always wait, unless its leader payoff exceeds the return from being a follower.

Now consider Corollary 2(3). This part captures a very different type of scenario than the equilibria described above. To garner the intuition for these cases, with the help of Figure 4, first consider Corollary 2(3b). As illustrated in the top panel of the Figure, if the game reaches t_1^* without entry, firm 1 would not enter at this time; rather it has an incentive to wait and enter at $\widehat{T}_1$. Understanding firm 1's incentive, as $L_2(t_1^*)$ is below its historical maximum, firm 2 has an incentive to preempt and enter at t_2^*.[16] Consequently, there is a unique equilibrium with firm 2 entering at t_2^*.

Note that this example satisfies the condition $L_1(t_1^*) < F_1(t_1^*)$. However, in general any sign between $L_1(t_1^*)$ and $F_1(t_1^*)$ is possible. To illustrate this, consider the example shown in Figure 4(b), in which firm 1 has a leader advantage (after $\widetilde{T}_1$) whereas there is a second-mover advantage for firm 2. This example is equivalent to the example presented in Figure 2C in Katz and Shapiro (1987). Employing the same logic as in the example in Figure 4(a), if the game were to reach t_1^* without entry, firm 1 would have an incentive to wait and only enter at $\widehat{T}_1$. At this time in the game, there is no credible way firm 2 can prevent firm 1 from waiting, as firm 2's follower payoff exceeds its return as a leader. Anticipating this, firm 2 has an incentive to preempt; there is a unique equilibrium with firm 2 entering at t_2^* (its historical maximum payoff as leader). As noted by Katz and Shapiro (1987),

[16] In Figure 4(a) $A_1(0) = (0, t_1^*]$, where t_1^* is determined by historical maximum of L_2. On the other hand, $A_2(0) = (0, t_2^*]$.

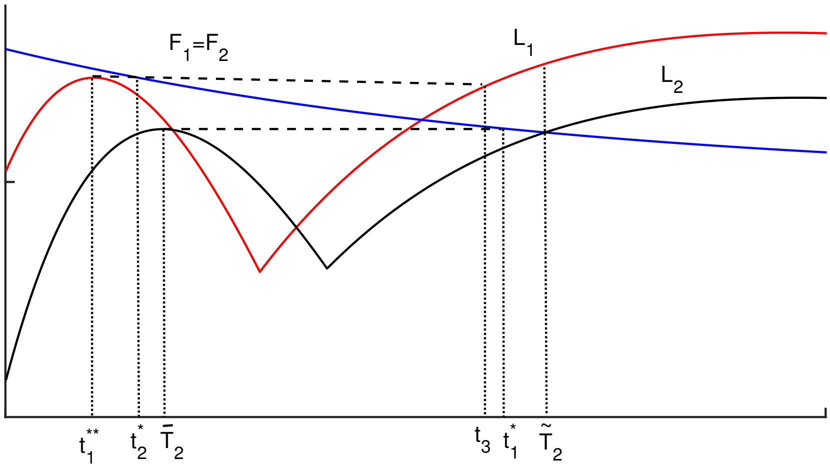

Figure 6: Example of blocking entry when both leader's payoffs can be multi-peaked functions

however, there is a complication that there does not exist an equilibrium with pure strategies for a truncated subgame on $[\widetilde{T}_1, T_2^{**}]$. This issue is discussed in Katz and Shapiro (1987), in footnote 15. One solution is to resort to mixed strategies over this range. Another possibility is to augment the equilibrium concept so as to require that either firm does not play dominated strategies. This caveat ensures entry by firm 2 at t_2^*, regardless of the strategy adopted by firm 1 after this time.

This issue of non-existence of equilibria is exacerbated further in Corollary 2(3a), illustrated in Figure 5. In this case is no pure-strategy SPE. Critical to this non-existence outcome is that $t_2^* < \widehat{T}_2$. The intuition for this result is as follows. In this scenario, recall that $A_1(t_1^*, T) \neq \emptyset$. This means that if the game reaches t_1^* without entry, firm 1 will have an incentive to delay entry further. Anticipating this, firm 2 would consider preemption; a candidate for preemption would be at a time at which its leader payoff is maximized, $\widehat{T}_2$. But in this example $t_2^* < \widehat{T}_2$, which raises the problem of existence. At $\widehat{T}_2$ firm 1's best response to entry by firm 2 is to preempt, as $L_1(\widehat{T}_2) > F_1(\widehat{T}_2)$. Firm 2 would prefer to follow if firm 1 enters, but if it does so, firm 1 would also have an incentive to wait, as its leader payoff is increasing at this time. Hence, there is no combination of best-response pure strategies, as formally captured by Corollary 2(3a).

Before leaving Corollary 2, it is worth noting that if any equilibrium exists, entry by the market leader always occurs at t_1^* or at t_2^*, or at both times (which could of course coincide). While not ensuring uniqueness, it does indicate that at most there are two entry times feasible in equilibrium.

To finish our discussion in this Section, consider the full set of possibilities covered in Proposition 2. The above discussion of Corollaries 1 and 2 centres the cases captured by Proposition 2(1), (2), (3a) and (3b). A novel element of our analysis relates to Proposition 2(3c). Let us highlight the intuition of this case with the assistance of the following example, illustrated in Figure 6. Firstly, in Figure 6 note that $A_1(0) = (0, t_1^{**}] \cup (t_3, t_1^*]$. On the other hand, $A_2(0) = (0, t_2^*]$. To determine the equilibrium entry time, we iterate backwards from the latest possible candidate entry date. If game has reached t_1^* without entry, firm 1 would rather wait and enter later at $\widetilde{T}_2$ than to lead at t_1^*. Anticipating this incentive for firm 1 to delay at t_1^*, firm 2 would prefer to preempt this outcome; from $A_2(0) = (0, t_2^*]$, a candidate entry time is t_2^*. However, note the structure of payoffs at t_2^*; as $F_1(t_2^*) > L_1(t_2^*)$ waiting dominates entry for firm 1, whereas firm 2's leader payoff is also increasing if it waits ($t_2^* < \bar{T}_2$). Consequently, if the game reaches t_2^* without entry, firm 2 would prefer to wait and only at $\bar{T}_2$. Of course, firm 1 will anticipate its fate if the game reaches t_2^*, giving it an incentive to preempt by entering even earlier at t_1^{**}. This is because its payoff as a follower between t_2^* and $\bar{T}_2$ is less than $L_1(t_1^{**})$. In this scenario, anticipating that it will be preempted, firm 1 enters even early in order to block its rival's preemption attempt. That is, anticipating that it will be preempted by firm 2 ($\widetilde{T}_2$), firm 1 blocks preemption by entering even earlier, in this case at t_1^{**}.

This is a new result, and we denoted such a situation as a *blocking entry* equilibrium. As noted in the introduction, this could relate to a firm prematurely launching a new smartphone, or cobbling together an updated release, in an attempt to block a rival's future entry that would have itself been, in its own right, a preemptive market entry. The *blocking entry* equilibrium is possible only in a mixed case covered in Proposition 2(3), in which the incentive to leader or follow reverses between the firms due to non-monotonic leader payoff functions. This example also illustrates that in the general case covered by the Proposition 2, entry times in equilibrium can differ from t_1^* and t_2^*.

3.3 Example of two cost-decreasing technologies

To provide some further intuition for the main results in the paper, and to allow for a closer comparison with the previous literature, we construct the following modification of Katz and Shapiro (1987). Essentially, we augment their example to allow for more than one potential innovation that firms can put into practice. As noted in the Introduction, firms are often faced with the choice between two or more alternative technologies. Examples of competing technologies for tablets, phone handsets and computer hardware come to mind, but a similar choice often has to be made when considering adopting cost-reducing technologies. Each technology will typically come with its own advantages. Moreover, technologies do not necessarily get developed at the same time or mature at the same rate.

Consequently, one technology might be preferred at by an early adopter, but a later entrant might well opt for an alternative as the relative advantages of the technologies change over time.

Here, we completely characterize all SPE of this two-innovation game using the algorithm outlined in this paper. By doing so, we show how an augmented example of Katz and Shapiro (1987) can provide micro-foundations for non-monotonic leader payoff functions, non-increasing follower payoffs, and for the equilibrium with *blocking entry*, as illustrated in Figure 6 and discussed above.

Consider the case when two firms are contemplating when to upgrade to a new technology, which they can implement at some time $t_i \in [0, \infty)$, for $i = 1, 2$. Each firm can choose to implement one of the two options $k = 1, 2$ available. For each firm, the old (null) technology generates a flow of profit normalized to zero; that is, $\pi_i^0 = 0$, $i = 1, 2$. After adoption, the new technology k affords firm i a flow of profit $\pi_{ik}^i > 0$. Subsequent to firm i adopting technology k, firm j's earns a flow profits of $\pi_{jk}^i > 0$. Finally, after firm i's adoption of k, industry profits are given by $\pi_k^i = \pi_{jk}^i + \pi_{ik}^i$.

The payoffs are discounted by a common discount factor e^{-rt}, so that the net-present value of profits for the leader (firm i) entering at t_i with technology k is:

$$L_i(t_i, k) = \int_{t_i}^{\infty} e^{-rt} \pi_{ik}^i dt - K^L(t_i, k) = \frac{e^{-rt_i}}{r} \pi_{ik}^i - K^L(t_i, k). \tag{5}$$

Here, we use the exponentially declining development cost function, $K(t_i, k) = K_0 e^{\lambda_k t_i} + K_{ik}$, with $\lambda_k > r$.

Similarly, the payoff to firm i if firm j enters with technology k at t_j is:

$$F_i(t_j, k) = \frac{e^{-rt_j}}{r} \pi_{ik}^j - K^F(t_i, k). \tag{6}$$

As firm i maximizes its payoff, the net-present value of profits for the leader (firm i) entering at t_i with the best technology available is:

$$L_i(t_i) = \max_{k=1,2} \left[\frac{e^{-rt_i}}{r} \pi_{ik}^i - K^L(t_i, k) \right]. \tag{7}$$

For simplicity, let us assume that the payoff to firm i if firm j wins with technology k is independent of k; that is,

$$F_i(t_j) = F_i(t_j, 1) = F_i(t_j, 2). \tag{8}$$

This means that for both firms $\pi_{i1}^j = \pi_{i2}^j$ and $K^F(t_i, 1) = K^F(t_i, 2)$; a follower earns the same level of profit, regardless as to the technology adopted by the market leader.

The market demand in each period is 1 unit at a constant price of 1. We assume that firms share the market equally; the profits before and after entry are

$$\pi_i^0 = (1 - c_i^0)/2, \;\; \pi_{ik}^i = (1 - c_{ik}^i)/2, \;\; \pi_{ik}^j = (1 - c_{ik}^j)/2,$$

where c_i^0, c_{ik}^i and c_{ik}^j are the costs corresponding to the old and new technology cases, respectively.

Several points are worth noting here in relation to this example and the analysis of the model above. Firstly, the follower's payoff function $F_i(t_j)$ is a decreasing function of the leader's entry and the parameters ensure that the payoffs are always positive. This means that Assumptions 5 and 6 are satisfied, and that we can apply our framework. Secondly, the three curves – $L_1(t)$, $L_2(t)$, and $F_1(t) = F_2(t)$ – in Figure 6 are all constructed using equations (7) - (8). In this way, we are able to construct an entry game with the characteristics of our model, with only a slight augmentation of an example in the literature. Thirdly, this example illustrates the case in which there is a unique pure-strategy SPE with blocking entry – a situation not previously considered. □

4 Impact of innovation policies

Proposition 3. *As a result of improvement of leader payoff (follower payoff), in the new SPE of the two-player symmetric timing game, the entry time can only stay the same or decrease (increase).*

Payoffs from innovation can be influenced by small (exogenous) changes in the environment. Policy makers regularly try to influence the incentive to innovate through subtle adjustments in subsidies, concessions or tax breaks for innovators. Typically the objective of these incentives is to bring forward or to encourage technological advancement. However, these policies can at times create unexpected or even perverse incentives in terms of the timing of innovation due to the strategic interaction of asymmetric oligopolists. The analysis in this section highlights the ambiguous and, at times, counter-intuitive impact innovation policy (or more broadly, changes in the leader and follower payoff functions) can have on technological advancement. To do so, in this section we consider the effects of changes to payoff functions.

By way of comparison, first consider the symmetric game in which both firms have the same potential payoffs, given the leader's time of entry, as in Smirnov and Wait (2015). If there is an increase in the return enjoyed by a leader (the leader curve shifts up), in equilibrium this change can only result in entry occurring at the same time, or early. To see this, revisit the examples shown in Figure 1. If the leader payoff function shifts up vertically, entry can never occur at a later time in the new equilibrium, relative to what it would have been without the change. Entry will occur early if there is a small vertical upwards shift in the $L(t)$ in the top panel of Figure 1, as the intersection of $L(t)$ and $F(t)$ occurs at an early time than prior to the change. On the other hand, as illustrated in the bottom of the Figure, the equilibrium entry time is unchanged if there is a small upwards shift in $L(t)$, as t^* still maximizes the leader's payoff, and this determines when the leader enters. An analogous argument can be made for a policy that increases the payoff for the follower over all t. If there is an increase in the follower payoff: (i) entry time in equilibrium will be unchanged (for example, in the second-mover advantage equilibrium in which entry is determined by the maximum of the leader payoff function, which could apply for a small increase in $F(t)$ in the bottom panel of Figure 1); or (ii) it will be at a later time (in a preemptive equilibrium, when entry occurs at the first intersection from below of the leader curve with the follower payoff function, as would be the case for a small constant increase in $F(t)$ in the top panel of Figure 1). An important driver of this result is the assumption that the follower's payoff is non-increasing in the leader's entry time.

This intuition is summarized in the proposition below.

Proposition 4. *As a result of improvement of leader payoff (follower payoff), in the new SPE of the two-player symmetric timing game, the entry time will be early (later) or remain unchanged.*

It is tempting to let this unambiguous prediction and the underlying intuition to guide innovation policy. But as it turns out, the more plausible asymmetric game in which the leader and follower payoff functions can differ between firms is far more complicated. To analyze the asymmetric game first focus on a change to the leader payoff curves. Explicitly, consider the situation when firm i costs $K_i^L(t_i, k)$ for both $k = 1, 2$ decrease by Δ, as a result of an unexpected taxation concession for the market leader.[17] As a result, firm i's new leader net-present value curve increased by Δ; that is, $L_i^{new}(t_1) = L_i^{old}(t_1) + \Delta$. To isolate the impact of this change assume that all other curves stay the same.

The following result characterizes how the timing of entry is impacted in SPE of the game with asymmetric payoff functions by an improvement in one of the leader payoff curves.

Result 1. *If there is an improvement in firm 1's (firm 2's) leader payoff, the time of entry in the new SPE of the two-player asymmetric timing game will be unchanged, or it will be at a later (early) date.*

Proof: See Appendix A.

[17] A change in the leader's payoff like this could also arise due to an exogenous advance in technology.

As shown previously, there are at most two equilibria of our timing game – which will involve entry at either t_2^* to t_1^*, or both.[18] In this case, increasing $L_1(t)$ can only switch entry from t_2^* or t_1^*. The converse is true for a vertical shift up of $L_2(t)$; such a change can only cause a switch in the equilibrium time of entry from t_1^* to t_2^*. Consequently, shifting one or other of the leader payoff functions through a policy or tax break can have opposite affect on the time of entry.

Consider now the impact of small exogenous changes to the follower payoff curves. Specifically, consider the situation when firm i costs $K_i^F(t_i, k)$ for both $k = 1, 2$ are decreased by Δ (as a result of a tax concession, a technological improvement, and so on). This means firm i's new follower net-present value curve has increased by Δ; that is, $F_i^{new}(t_1) = F_i^{old}(t_1) + \Delta$. As before, to aid in our analysis all other curves remain unchanged.

The following result characterizes changes in the timing of entry in the pure-strategy SPE with asymmetric firms that arises as a result of an improvement of either of the follower payoff functions.

Result 2. *An increase in the follower payoff function for firm 1 (firm 2), results in the entry time in the new SPE with asymmetric firms to remain unchanged or be early (delayed).*

Proof: See Appendix A.

Again, this result relies on the whether or not the timing of entry switches between one of the two candidate equilibria, or if it remains unchanged. Remember that there are at most two equilibria of the game; one equilibria involves firm 1 entering, whereas two potential equilibria involve firm 2 entering. Given this, increasing the follower payoff for firm 1 (firm 2) can only result in a switch in the entry time from t_1^* to t_2^*, or that it remain at t_1^*. Using similar reasoning, an increase in the follower payoff for firm 2 can only resulting in a switch from t_2^* to t_1^* or, alternatively, that the time of entry remain at t_2^*

Herein lies a complication for a policy makers. Any exogenous change, possibly arising from an innovation policy implemented by the government, in which both leader payoff curves improve has an *a priori* ambiguous impact on the time of entry. A similar point can be made for changes that enhance the returns experienced by both firms as second entrants – without knowing the specific payoff functions, it is not possible to ascertain how the time of innovation will be impacted by an policy that reduces the cost of innovation. This is a cautionary tale for policy makers. It also is a possible explanation for the invariance or perverse response of a potential innovator to policies designed to enhance the take-up of new technology. The analysis of the two Results above are summarized in the following proposition.

[18] For ease of exposition, let us not consider the case when entry occurs at another time, as in the *blocking entry* equilibrium. This restriction is for the discussion only. These result holds for all equilibria of the game.

Proposition 5. *An improvement in both player's leader payoffs (follower payoffs) has an ambiguous impact on the timing of entry in the SPE of the two-player asymmetric timing game.*

In essence, a symmetric innovation policy applied to asymmetric firms has an ambiguous effect on the timing of entry.

5 Concluding comments

The decision when to launch a product or implement a new production process is a critical question for many firms; it can help determine profit, firm survival and the shape of markets. More generally, it drives economic development. Given its importance, innovation has received a great deal of attention from economists, notably the seminal paper of Fudenberg and Tirole (1985). The previous literature has often assumed that firms are symmetric in terms of their potential to exploit the new technology, and that all relevant technologies are available at the start of the game. Both of these assumptions are important, as some firms are better placed to take advantage of an innovative opportunity than others, and not all technologies (or market entry opportunities) are available immediately. Adapting Fudenberg and Tirole (1985), Katz and Shapiro (1987) and Smirnov and Wait (2015), amongst others, we develop a novel market-entry game with asymmetric payoffs between firms. Moreover, our model allows for the complication that a new technology might only become available to a firm after some waiting period. To capture this, we allow the leader's payoff function to be non-monotonic and even multi-peaked. Indeed, the main restrictions we place on the payoff functions is that they are continuous and that the follower's payoff is non-increasing in the time of the leader's entry.

The first contribution of the paper is a technical one. We provide a method to solve for the equilibrium entry time (or times) in pure strategies. We show that, provided it exists, there are at most two entry times in equilibrium. Given its generality, our setup allows us to study several new phenomena associated to market entry not possible in the previous literature. For example, allowing for firm asymmetry generates various possibilities, such as first- and second-mover advantages co-existing at the same time for either of the firms. We also show that *blocking entry* is possible, in which a firm, fearing being preempted itself, comes into the market even earlier, effectively preempting the preemptor. Allowing for asymmetries between firms also highlights the difficulties of intervening in the market in order to influence the take-up of new technologies. We show, for example, that increasing the payoff to both firms to be a market leader has an ambiguous effect on the timing of entry in equilibrium.

6 Appendix A

Proof of Proposition 2

This proof consists of five parts: A, B, C, D, and E. In Part A we show that all SPE with positive entry times must belong to either $A_1(0,T)$ if firm 1 enters first or $A_2(0,T)$ if firm 2 enters first. In Part B we prove that for $i = 1, 2$ there exists a unique t_i^*, given by (3), at which either $L_i(t)$ is maximized over $A_i(0,T)$ or $t_i^* = 0$ when $A_i(0,T) = \emptyset$. Part C shows that firm 1 entering at t_1^* is a unique SPE as it delivers the highest possible equilibrium payoffs to both the leader and the follower when $A_1(t_1^*, T) = \emptyset$ and $L_1(t_1^*) > F_1(t_1^*)$. Part D proves that if $A_1(t_1^*, T) = \emptyset$ and $L_1(t_1^*) \leq F_1(t_1^*)$ then there are two SPE with firm 1 entering at $t = t_1^*$ and firm 2 entering at $t = t_2^*$. Finally, part E considers the scenario when $A_1(t_1^*, T) \neq \emptyset$.

(A) As a preliminary step, let us prove all SPE with positive entry times must belong to either $A_1(0,T)$ if firm 1 enters first or $A_2(0,T)$ if firm 2 enters first. Assume, on the contrary, that there is an SPE with a positive entry time $t_i^* \notin A_i(0,T)$. It must be the case that both NPL and NPF conditions are satisfied. If condition NPL is not satisfied, the leader (player i) will have an incentive to enter earlier at τ. On the other hand, if condition NPF is not satisfied, the follower (player j) will have an incentive to preempt the leader (player i) and enter slightly earlier, as in Fudenberg and Tirole (1985). Neither of these situations are possible in equilibrium. Consequently, there is a contradiction, proving the statement that all SPE with positive entry times must belong to either $A_1(0,T)$ if firm 1 enters first or $A_2(0,T)$ if firm 2 enters first.

(B) Next, let us prove that t_i^* for $i = 1, 2$ is given by (3). Specifically, there exists a unique t_i^* at which either $L_i(t)$ is maximized over $A_i(0,T)$ or $t_i^* = 0$ when $A_i(0,T) = \emptyset$. When $A_i(0,T) = \emptyset$, entering at $t_i^* > 0$ can not be an SPE; so the only potential entry time for player i is $t_i^* = 0$.

Now consider the situation when $A_i(0,T)$ is not empty. Let us prove the existence of the solution to this problem of maximizing $L_i(t)$ over $A_i(0,T)$ when $A_i(0,T) \neq \emptyset$. Note that set $A_i(0,T)$ is bounded because $\widehat{T_i}$ is finite, where $\widehat{T_i}$ is the time at which $L_i(t)$ reaches its global maximum (Assumption 4). We need to show that set $A_i(0,T)$ always contains its supremum. Assume that it does not. This means that there is a sequence $\{t_k\}$ contained in $A_i(0,T)$ that converges to some limit t_i^* that is not contained in set $A_i(0,T)$. This requires that either NPL or NPF is not satisfied for t_i^*. As sequence $\{t_k\}$ belongs to $A_i(0,T)$, it means that both NPL and NPF hold for sequence $\{t_k\}$. As both $L_i(t)$ and $F_j(t)$ are continuous functions, it means t_i^* also belongs to $A_i(0,T)$. This leads to a contradiction, proving existence.

The uniqueness follows immediately from the way set $A_i(0,T)$ is constructed. If two entry times were to maximize $L_i(t)$ over $A_i(0,T)$, then the later time would not belong to $A_i(0,T)$.

Next, let us show that if $t_i^* = \arg\max_{t \in A_i(0,T)} L_i(t)$, it is also the case that $t_i^* = \arg\max_t A_i(0,T)$ when $A_i(0,T) \neq \emptyset$. Assume the opposite that $t_i^* \neq \arg\max_t A_i(0,T)$. If $t_i^* < \arg\max_t A_i(0,T)$, then t_i^* does not maximize the leader's payoff over $A_i(0,T)$. If $t_i^* > \arg\max_t A_i(0,T)$, t_i^* does not belong to $A_i(0,T)$. Both situations lead to a contradiction. We have now shown that $t_i^* = \arg\max_t A_i(0,T)$, concluding the proof of Part B.

(C) Next, we prove that firm 1 entering at t_1^* given by (3) is a unique SPE when $A_1(t_1^*,T) = \emptyset$ and $L_1(t_1^*) > F_1(t_1^*)$. Note that in this case $t_1^* > t_2^*$. First, in part A we proved that there is no equilibrium with entry time $\tau > t_1^*$.

Second, let us prove that firm 1 entering at t_1^* is an SPE. In part B we proved that t_1^* maximizes $L_1(t)$ over $A_1(0,T)$. This means the leader gets the highest possible equilibrium payoff and has no incentive to deviate. The follower has no incentive to deviate as well as t_1^* belongs to $A_1(0,T)$ and $A_1(t_1^*,T) = \emptyset$.

Third, let us prove that there is no equilibrium with entry time of firm 1 $\tau < t_1^*$. As $t_1^* \in A_1(0,T)$, entering at $t < t_1^*$ is strictly dominated by entering at t_1^*.

Third, let us prove that firm 1 entering at t_1^* is an SPE. In part B we proved that t_1^* maximizes $L_1(t)$ over $A_1(0,T)$. This means the leader gets the highest possible equilibrium payoff and has no incentive to deviate. The follower has no incentive to deviate as well as t_1^* belongs to $A_1(0,T)$ and $A_1(t_1^*,T) = \emptyset$.

Finally, let us prove that there is no SPE with firm 2 entering at any t. Part A guarantees that entering at $t > t_2^*$ can not be an SPE. The follower also has no incentive to enter at $t \leq t_2^* < t_1^*$ as t_1^* belongs to $A_1(0,T)$, which means NPF condition is satisfied and for firm 2 waiting dominates entering.

(D) Let us prove that if $A_1(t_1^*,T) = \emptyset$ and $L_1(t_1^*) \leq F_1(t_1^*)$ then there are two SPE with firm 1 entering at $t = t_1^*$ and firm 2 entering at $t = t_2^*$. Consider an SPE where firm i is the leader and firm j is the follower ($i, j = 1, 2$ and $i \neq j$).

First, given $t_i^* \in A_i(0,T)$, if the follower deviates by entering at some time $\tau < t_i^*$, it will get a payoff of $L_j(\tau) < F_j(t_i^*)$. If it deviates by entering at t_i^*, it will get a payoff of $(L_j(t_i^*) + F_j(t_i^*))/2$, which is less than $F_j(t_i^*)$. If the follower enters at $t > t_i^*$, there will be no change to the equilibrium outcome. Consequently, there is no profitable deviation for the follower.

Second, in part A we proved that there is no equilibrium with the leader entering at $\tau > t_i^*$. Given $t_i^* \in A_i(0,T)$, if the leader deviates by entering earlier at some time $\tau < t_i^*$, it will get a payoff of $L_i(\tau) < L_i(t^*)$. There is no profitable deviation for the leader.

There is no other equilibria as entering at t_i^* dominates entering at any other time. Consequently, we have proved that there are two equilibria.

(E) Let us first consider case (b) when $A_1(t_1^*,T) \neq \emptyset$, $A_2(t_2^*,t_1^*) = \emptyset$ and $t_2^* = \bar{T}_2$.

In this scenario in comparison to (D), $A_1(t_1^*, T) \neq \emptyset$. This means firm 1 entering at $t = t_1^*$ can not be an equilibrium as at t_1^* firm 1 has an incentive to wait; firm 2 will have incentive to block firm 1's entry. Note also that for the same reason as in part (D), entering by either firm i at any other time than t_i^* can not be an SPE. Consequently, there is only the remaining SPE with firm 2 entering at $t = t_2^*$.

Consider instead case (c) when $A_1(t_1^*, T) \neq \emptyset$ and $A_2(t_2^*, t_1^*) \neq \emptyset$. In this case in comparison with case (b), neither entering by firm 1 at t_1^* nor entering by firm 2 at t_2^* can be an equilibrium. Firm 1 has strong incentives to enter even earlier (and block firm 2's entry) as firm 2 has incentives to wait at t_2^*. Consequently, there is an SPE with firm 1 entering at t_1^{**}. This equilibrium is unique for the same reasons as in case (b).

Finally consider 3 (a) when $A_1(t_1^*, T) \neq \emptyset$, $A_2(t_2^*, t_1^*) = \emptyset$ and $t_2^* < \bar{T}_2$. In this case it must be that $t_2^* = \widetilde{T}_1$, which in turn imply that $L_1(\widehat{T}_2) > F_1(\widehat{T}_2)$ and $L_2(\widehat{T}_2) < F_2(\widehat{T}_2)$. There is no pure strategy equilibrium as at $\widehat{T}_2$ firm 1's best response to entry by firm 2 is to enter slightly earlier. Firm 2 would prefer to follow if firm 1 enters, but if it does so, firm 1 would also have an incentive to wait, as its leader's payoff is increasing at this time. This observation concludes the proof of the Proposition. □

Proof of Result 1

First, consider the change in firm 1's leader payoff. If the original SPE satisfies $A_1(t_1^*, T) = \emptyset$ and $L_1(t_1^*) > F_1(t_1^*)$ then there is a unique SPE with firm 1 entering at $t = t_1^*$. All technology improvement is collected by the first firm.

If the original SPE is of the second-mover advantage type, $A_1(t_1^*, T) = \emptyset$ and $L_1(t_1^*) \leq F_1(t_1^*)$, then the following two situations are possible. If $\Delta \leq L_1(t_1^*) - F_1(t_1^*)$ then both original second-mover advantage equilibria still exist. On the other hand, if $\Delta > L_1(t_1^*) - F_1(t_1^*)$ then only SPE with firm 1 entering at $t = t_1^*$ remains. Neither of these situations lead to decrease in the time of entry.

Consider now the scenario with $A_1(t_1^*, T) \neq \emptyset$.

Second, consider the change in firm 2's leader payoff.

This concludes the proof of this Result. □

Proof of Result 2

A similar logics applies as in Result 1. □

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