Information Acquisition with Subjective Waiting Costs

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November, 15, 2018

Motivation

- Decisions are not necessarily made on the basis of fixed prior information
- Acquiring information is costly, and the agent has to balance the benefits and costs
- Rational inattention (for example, Sims (2003))
- Additive information costs have been considered
- Costs for information acquisition are unobservable, and adhocracy of its modeling is problematic

- There are two approaches for identifying information costs
- State-dependent stochastic choice from menus
  - Caplin and Dean (2015)
  - A signal arrives $\Longrightarrow$ choice is made from menu $\Longrightarrow$ stochastic choice
- Preference over menus
  - de Oliveira, Denti, Mihm, and Ozbek (2017)
  - Menu choice $\Longrightarrow$ signal arrives $\Longrightarrow$ choice is made from menu

- There are some instances where implications of additive information costs are not intuitive
- State-dependent stochastic choice
  - Chambers, Liu, and Rehbeck (2018)
  - Non-additive costs, multiplicative costs
- Preference over menus
  - This paper
  - Multiplicative costs

Implications of additive information costs

- Two states $\Omega = \{\omega_1, \omega_2\}$
- Acts (x_1, x_2): monetary payoffs, linear utility (risk neutral)
- Suppose
$$\{(100, 0)\} \sim \{(0, 100)\} \sim \{(50, 50)\}$$
- The agent's prior over Ω is given by $(\frac{1}{2}, \frac{1}{2})$
- By preference for flexibility, he may exhibit
$$\{(100, 0), (0, 100)\} \sim \{(60, 60)\} \succ \{(100, 0)\} \sim \{(0, 100)\}$$
- Facing with the menu $\{(100, 0), (0, 100)\}$, the agent optimally solves costly information acquisition
- The marginal (net) benefit of acquiring this information structure is given as $60 - 50 = 10$

Additive Information Costs Representation: (de Oliveira, Denti, Mihm, and Ozbek (2017))

$$U(F) = \max_{\pi \in \Pi} \left\{ \int_{\Delta(\Omega)} \max_{f \in F} \left(\sum_{\omega \in \Omega} u(f(\omega)) p(\omega) \right) \mathrm{d}\pi(p) - c(\pi) \right\}$$

- Implies Translation Invariance

$$U(F + \theta) = U(F) + u(\theta)$$

- Implies

$$F \succsim G \iff F + \theta \succsim G + \theta$$

- Translation invariance implies for all $m > 0$,

$$\{(100+m, m)\} \sim \{(m, 100+m)\} \sim \{(50+m, 50+m)\},$$
$$\text{and } \{(100+m, m), (m, 100+m)\} \sim \{(60+m, 60+m)\}$$

- The marginal (net) benefit of acquiring information structure is still given as 10
- An optimal information structure is invariant between $\{(100, 0), (0, 100)\}$ and $\{(100+m, m), (m, 100+m)\}$
- m does not affect an incentive for costly information acquisition

- Costs for information acquisition may come from time delay of decision
- For large m, the significance of the state-dependent payoff of 100 relative to the constant payoff m seems to be diminished
- If the decision is delayed by information acquisition, the constant payoff m is also delayed and this cost from waiting becomes more significant when m is large
- For large m, the agent may become less willing to acquire a new information structure:

$$\{(60, 60)\} \sim \{(100, 0), (0, 100)\},$$

but

$$\{(60 + m, 60 + m)\} \succ \{(100 + m, m), (m, 100 + m)\}$$

Road Map

- Introduce formal model
- Behavioral foundations (Representation Theorem)
- Proof sketch
- Application

Primitives

- $\Omega = \{\omega_1, ..., \omega_n\}$: the (finite) objective state space
- X: outcomes, consisting of simple lotteries on a set of deterministic prizes
- $f : \Omega \to X$: an (Anscombe-Aumann) act
- $\mathcal{F}$: the set of all acts
- $F \subset \mathcal{F}$: a finite set of acts, called a menu
- $\mathbb{F}$: the set of all menus
- $\succsim$ over $\mathbb{F}$

Functional Form

- $\pi \in \Delta(\Delta(\Omega))$: A signal (or information structure) on Ω

Definition: Blackwell order

A signal $\pi \in \Delta(\Delta(\Omega))$ is Blackwell more informative than a signal $\rho \in \Delta(\Delta(\Omega))$, denoted $\pi \trianglerighteq \rho$, if

$$\int_{\Delta(\Omega)} \varphi(p)\, \mathrm{d}\pi(p) \geq \int_{\Delta(\Omega)} \varphi(p)\, \mathrm{d}\rho(p)$$

for every convex continuous function $\varphi : \Delta(\Omega) \to \mathbb{R}$

- Partial order on $\Delta(\Delta(\Omega))$

- For each π, the initial prior $p^{\pi} \in \Delta(\Omega)$ associated with π is defined as

$$p^{\pi}(\omega) = \int_{\Delta(\Omega)} p(\omega)\, \mathrm{d}\pi(p)$$

- Let $\Pi \subset \Delta(\Delta(\Omega))$ denote a set of subjectively possible signals

Definition: Discounting cost function

We say that $\beta : \Pi \to (0, 1]$ is a discounting cost function if

1. there exists $\overline{p} \in \Delta(\Omega)$ with $\beta(\delta_{\overline{p}}) = 1$
2. for all $\pi, \rho \in \Pi$, $\pi \trianglerighteq \rho \implies \beta(\pi) \leq \beta(\rho)$
3. for all $\pi \in \Pi$, $\beta(\pi) p^{\pi}(\omega) \leq \overline{p}(\omega)$ for all ω

- Alternative to (iii): $p^{\pi} = \overline{p}$ for all $\pi \in \Pi$

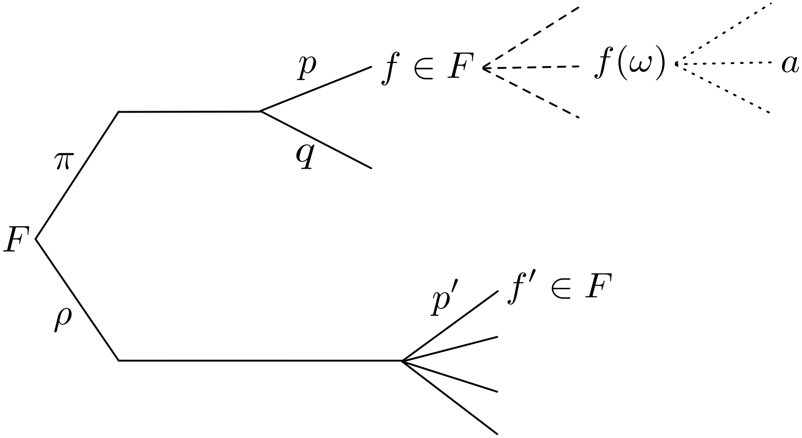
Definition: Optimal Waiting Representation

An Optimal Waiting Representation is a tuple $(u, \Pi, \beta, \overline{p})$, where

- $u : X \to \mathbb{R}_+$ is an unbounded expected utility function with $u(X) = [0, \infty)$,
- $\overline{p} \in \Delta(\Omega)$ is the initial prior,
- Π is the set of possible signals,
- $\beta : \Pi \to (0, 1]$ is a discounting cost function

such that $\succsim$ is represented by

$$U(F) = \max_{\pi \in \Pi} \left\{ \beta(\pi) \int_{\Delta(\Omega)} \max_{f \in F} \left(\sum_{\omega \in \Omega} u(f(\omega)) p(\omega) \right) \mathrm{d}\pi(p) \right\}$$


p
$f \in F$
$f(\omega)$
a
π
q
F
p'
$f' \in F$
ρ

- For any singleton menu $F = \{f\}$ and $\pi \in \Pi$,

$$\int_{\Delta(\Omega)} \max_{f \in F} \left(\sum_{\omega \in \Omega} u(f(\omega))p(\omega) \right) \mathrm{d}\pi(p) = \sum_{\Omega} u(f(\omega))p^{\pi}(\omega)$$

- By property (iii) of $\beta(\pi)$,

$$U(\{f\}) = \max_{\pi \in \Pi} \beta(\pi) \sum_{\Omega} u(f(\omega))p^{\pi}(\omega) = \sum_{\Omega} u(f(\omega))\overline{p}(\omega)$$

- Since a choice with commitment reflects the decision maker's initial belief, $\overline{p}$ is interpreted as an initial prior

- Let's come back to the motivating example:

$$\{(60+m, 60+m)\} \succ \{(100+m, m), (m, 100+m)\}$$

- Given an optimal waiting representation,

$$\begin{aligned}
& U(\{(100+m, m), (m, 100+m)\}) \\
= & \max_{\pi} \beta(\pi) b_{\{(100+m,m),(m,100+m)\}}(\pi) \\
= & \max_{\pi} \beta(\pi)(b_{\{(100,0),(0,100)\}}(\pi) + m) \\
= & \beta(\pi^*) b_{\{(100,0),(0,100)\}}(\pi^*) + \beta(\pi^*) m \\
\leq & \beta(\pi^*) b_{\{(100,0),(0,100)\}}(\pi^*) + m \\
\leq & \max_{\pi} \beta(\pi) b_{\{(100,0),(0,100)\}}(\pi) + m \\
= & U(\{(100,0),(0,100)\}) + m \\
= & U(\{(60,60)\}) + m \\
= & U(\{(60+m, 60+m)\})
\end{aligned}$$

- Discounting costs model (This paper):

$$U(F) = \max_{\pi \in \Pi} \beta(\pi) b_F^u(\pi)$$

- Additive costs model (de Oliveira, Denti, Mihm, and Ozbek (2017)):

$$U(F) = \max_{\pi \in \Pi} \{ b_F^u(\pi) - c(\pi) \}$$

- Constant costs model (Dillenberger, Lleras, Sadowski, and Takeoka (2014)):

$$U(F) = b_F^u(\pi^*)$$

Foundation: Axioms

Axiom: Order

$\succsim$ is complete and transitive

Axiom: Mixture Continuity

For all F, G, H, the following sets are closed:

$$\{\alpha \in [0,1] \,|\, \alpha F + (1-\alpha)G \succsim H\} \text{ and } \{\alpha \in [0,1] \,|\, H \succsim \alpha F + (1-\alpha)G\}$$

- there exists $x_* \in X$ such that $F \succsim \{x_*\}$ for all $F \in \mathbb{F}$

Axiom: Unboundedness

There are outcomes $x, y \in X$ with $\{x\} \succ \{y\} \succ \{x_*\}$ such that for all $\alpha \in (0,1)$, there is $z \in X$ satisfying either $\{y\} \succ \{\alpha z + (1-\alpha)x\}$ or $\{\alpha z + (1-\alpha)y\} \succ \{x\}$

Axiom: Preference for flexibility

For all F, G,

$$G \subset F \implies F \succsim G$$

Axiom: Dominance

For all F and acts g, if there exists $f \in F$ with $f(\omega) \succsim g(\omega)$ for all $\omega \in \Omega$, then $F \sim F \cup \{g\}$

Mixture of menus

For all F, G and $\alpha \in (0,1)$,

$$\alpha F + (1-\alpha)G = \{\alpha f + (1-\alpha)g \mid f \in F, g \in G\}$$

Axiom: Independence

For all F, G, H, and $\alpha \in (0,1)$

$$F \succsim G \iff \alpha F + (1-\alpha)H \succsim \alpha G + (1-\alpha)H$$

(Dillenberger, Lleras, Sadowski, and Takeoka (2014)

$\succsim$ satisfies the above axioms if and only if it admits a constant costs representation

Axiom: Singleton Independence

For all acts f, g, h, and $\alpha \in (0,1)$

$$\{f\} \succsim \{g\} \iff \alpha\{f\} + (1-\alpha)\{h\} \succsim \alpha\{g\} + (1-\alpha)\{h\}$$

Axiom: Aversion to Contingent Planning

For all F, G and $\alpha \in (0,1)$,

$$F \sim G \implies F \succsim \alpha F + (1-\alpha)G.$$

- Convexity
- For example, suppose

$$\{(100,0),(0,100)\} \sim \{(80,80)\}$$

- If $\succsim$ satisfies Independence,

$$\{(100,0),(0,100)\} \sim \{(90,40),(40,90)\}$$

Axiom: Translation Invariance

For all F, G and a feasible translation θ on X,

$$F \succsim G \iff F + \theta \succsim G + \theta$$

- If $\{(60, 60)\} \sim \{(100, 0), (0, 100)\}$, then

$$\{(60 + m, 60 + m)\} \sim \{(100 + m, m), (m, 100 + m)\}$$

- Adding constants does not affect an incentive for information acquisition

de Oliveira, Denti, Mihm, and Ozbek (2017)

$\succsim$ on $\mathbb{F}$ satisfies the above axioms if and only if it admits an additive costs representation

Axiom: Worst Independence

For all F, G and $\alpha \in (0,1)$,

$$F \succsim G \iff \alpha F + (1-\alpha)\{x_*\} \succsim \alpha G + (1-\alpha)\{x_*\}$$

- homotheticity
- If $\{(60,60)\} \sim \{(100,0),(0,100)\}$, then

$$\{(60\alpha,60\alpha)\} \sim \{(100\alpha,0),(0,100\alpha)\}$$

- Scaling up or down does not affect an incentive for information acquisition

Representation Theorem

Theorem

$\succsim$ on $\mathbb{F}$ satisfies Order, Mixture Continuity, Unboundedness, Preference for Flexibility, Dominance, Singleton Independence, Aversion to Contingent Planning, Worst Independence if and only if it admits an optimal waiting representation

Proof Sketch

- There exists an EU function $u : X \to \mathbb{R}$ with unbounded range and a prior $\overline{p} \in \Delta(\Omega)$ such that $\succsim$ over $\mathcal{F}$ is represented by

$$U(f) = \sum_{\Omega} u(f(\omega))\overline{p}(\omega)$$

- Without loss of generality, $u(x_*) = 0$.
- For all F, there exists $x_F \in X$ such that $x_F \sim F$
- $U : \mathcal{F} \to \mathbb{R}$ is extended to $\mathbb{F}$ by $U(F) = U(x_F)$

- For any $F \in \mathbb{F}$ and any $p \in \Delta(\Omega)$, let

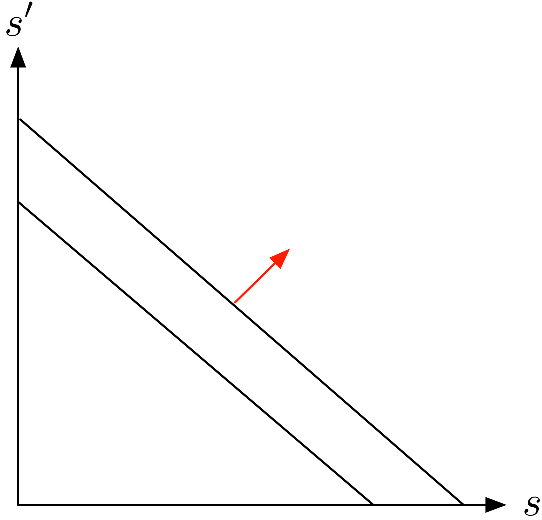
$$\varphi_F(p) = \max_{f \in F} \sum_{\Omega} u(f(\omega))p(\omega)$$

- The support function of F: $\varphi_F : \Delta(\Omega) \to \mathbb{R}$
- $\Phi_{\mathbb{F}} = \{\varphi_F \mid F \in \mathbb{F}\} \subset C(\Delta(\Omega))$
- $\varphi_F = \varphi_G \implies F \sim G$
- Define the functional $V : \Phi_{\mathbb{F}} \to \mathbb{R}$ by $V(\varphi_F) = U(F)$
- If V is linear,

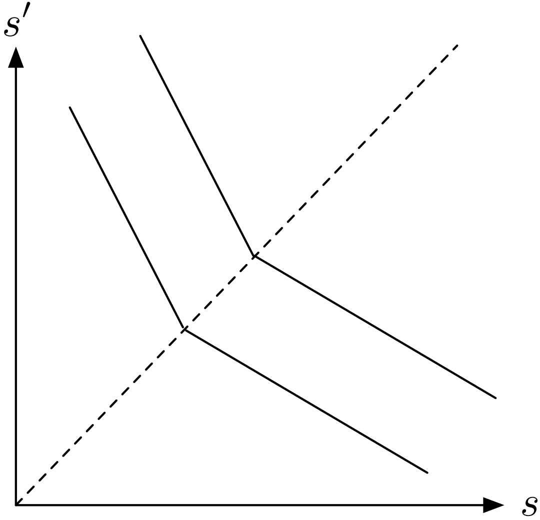
$$V(\varphi_F) = \int_{\Delta(\Omega)} \varphi_F(p) \, \mathrm{d}\pi^*(p)$$

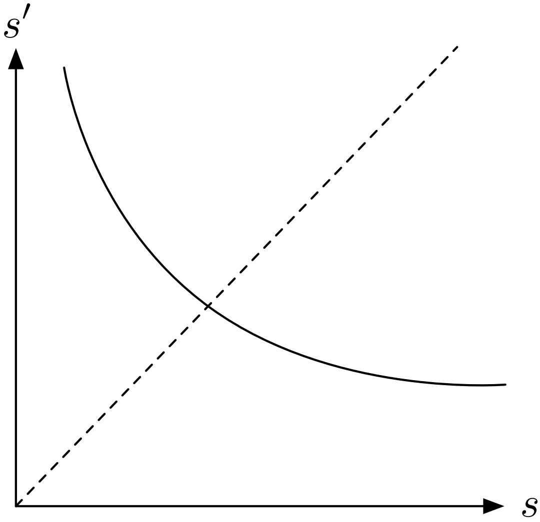
$\implies$ Constant costs model

Subjective EU: Anscombe and Aumann (1963)



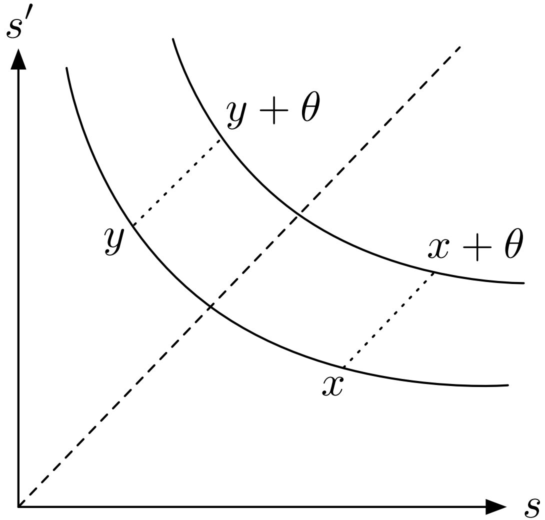
Maxmin EU: Gilboa and Schmeidler (1989)



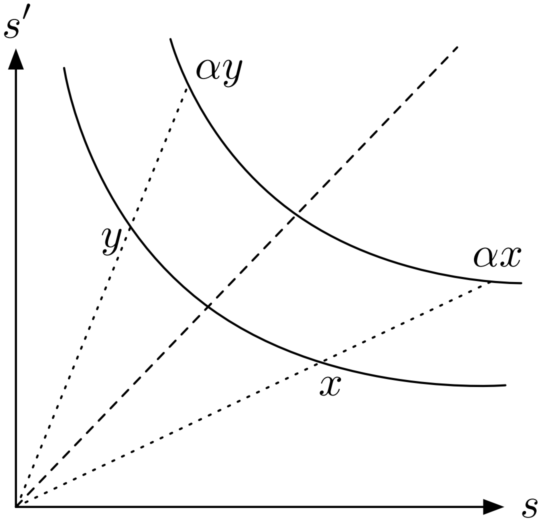


s'
s

Variational Preference: Maccheroni, Marinacci, and Rustichini (2006)

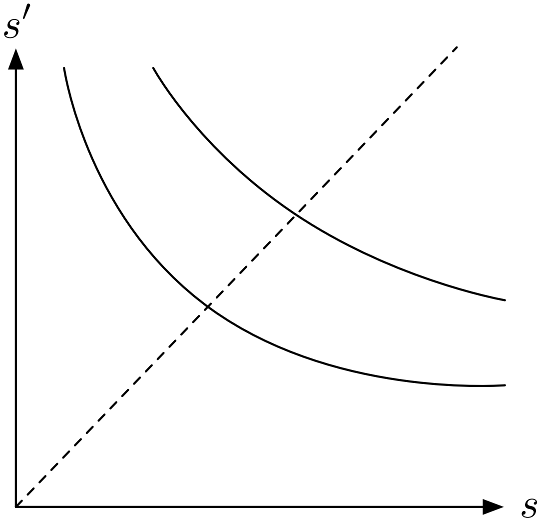


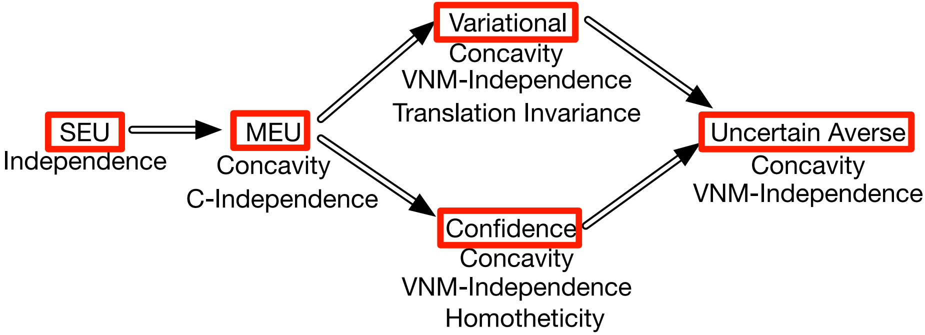
Confidence Preference: Chateauneuf and Faro (2009)



Uncertain Averse Preference:

Cerreia-Vioglio, Maccheroni, Marinacci, and Montrucchio (2011)




SEU
Independence
MEU
Concavity
C-Independence
Variational
Concavity
VNM-Independence
Translation Invariance
Confidence
Concavity
VNM-Independence
Homotheticity
Uncertain Averse
Concavity
VNM-Independence

- If U is a variational class,

$$U(f) = \min_{p \in \Delta(S)} \left\{ \int_S f(s)\,\mathrm{d}p(s) + c(p) \right\}$$

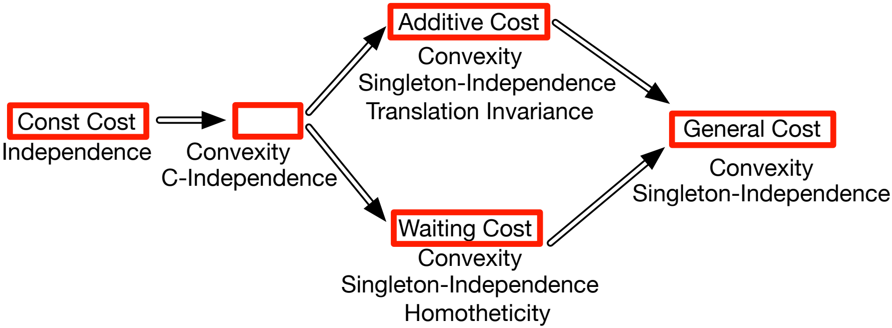
- If U is a confidence class,

$$U(f) = \min_{p \in \Delta(S)} \left\{ \beta(p) \int_S f(s)\,\mathrm{d}p(s) \right\}$$

- If U is an uncertain averse class,

$$U(f) = \min_{p \in \Delta(S)} G\left(\int_S f(s)\,\mathrm{d}p(s), p \right)$$

Preferences over Menus



- If V is a variational class,

$$V(\varphi_F) = \max_{\pi \in \Delta(\Delta(\Omega))} \left\{ \int_{\Delta(\Omega)} \varphi_F(p)\, \mathrm{d}\pi(p) - c(\pi) \right\}$$

$\Longrightarrow$ Additive costs model

- If V is a confidence class,

$$V(\varphi_F) = \max_{\pi \in \Delta(\Delta(\Omega))} \left\{ \beta(\pi) \int_{\Delta(\Omega)} \varphi_F(p)\, \mathrm{d}\pi(p) \right\}$$

$\Longrightarrow$ Waiting costs model

- If V is an uncertain averse class,

$$V(\varphi_F) = \max_{\pi \in \Delta(\Delta(\Omega))} G\left(\int_{\Delta(\Omega)} \varphi_F(p)\, \mathrm{d}\pi(p), \pi \right)$$

$\Longrightarrow$ General costs model

Application: Additive or discounting costs for information

- Cukierman (1980)
- The state space: $\Omega = \mathbb{R}$
- The prior: $\omega \sim N(\mu, 1/\tau)$
- Actions: $y \in \mathbb{R}$
- Payoffs: $u(y, \omega) = a\omega - b|\omega - y|$, $a > 0$, $b > 0$
- Signals: $s \sim N(\omega, 1/p)$
- The agent has an additive cost function such as

$$U(F) = \max_t \{b_F(t) - ct\},$$

where

$$b_F(t) = \int \max_{y \in F} \int u(y, \omega) \, \mathrm{d}p(\omega | s_1, \cdot, s_t) \, \mathrm{d}\sigma(s_1, \cdots, s_t)$$

- Cukierman (1980) shows that

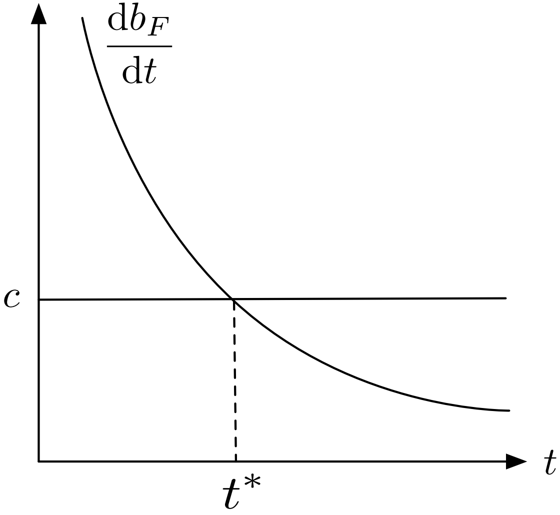
$$b_F(t) = a\mu - b\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\left(\frac{1}{\tau + tp}\right)^{\frac{1}{2}}$$

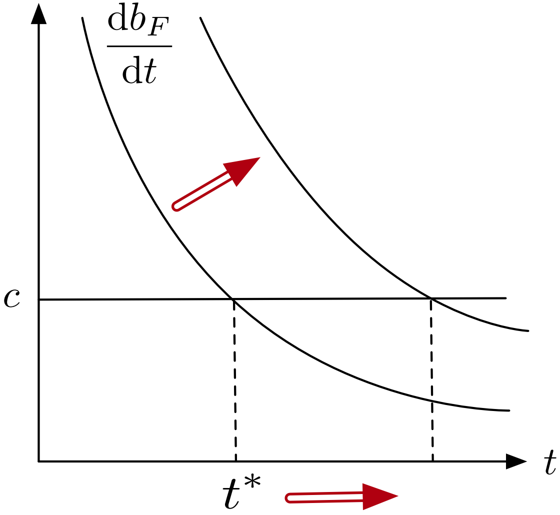
- By FOC, an optimal information acquisition is obtained by

$$\frac{\mathrm{d}b_F}{\mathrm{d}t}(t) = c$$

or

$$bp\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\left(\frac{1}{\tau + tp}\right)^{\frac{3}{2}} = c$$


$\frac{\mathrm{d}b_F}{\mathrm{d}t}$
c
t^*
t


$\frac{\mathrm{d}b_F}{\mathrm{d}t}$
c
t
t^*

Proposition (Cukierman (1980))

Assume the additive cost model:

1. An increase in the variance of the relevant stochastic variable decreases the quantity of current investment:

$$\tau \downarrow \Longrightarrow t^* \uparrow$$

2. An increase in the mean of the relevant stochastic variable is independent of the quantity of current investment:

$$\mu \uparrow \Longrightarrow t^* \text{ invariant}$$

- The agent has a discounting cost function such as

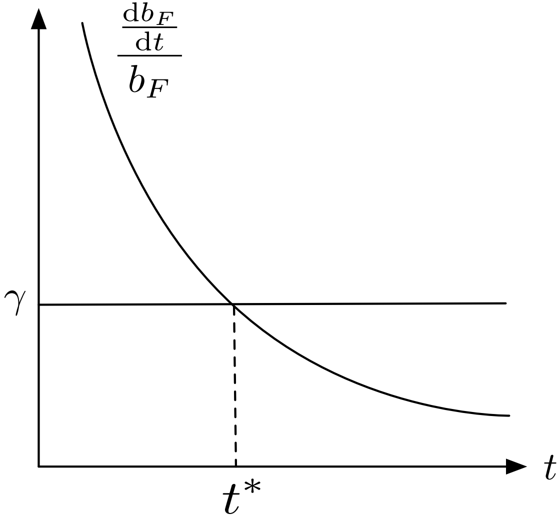
$$U(F) = \max_t e^{-\gamma t} b_F(t)$$

- By FOC, an optimal information acquisition is obtained by

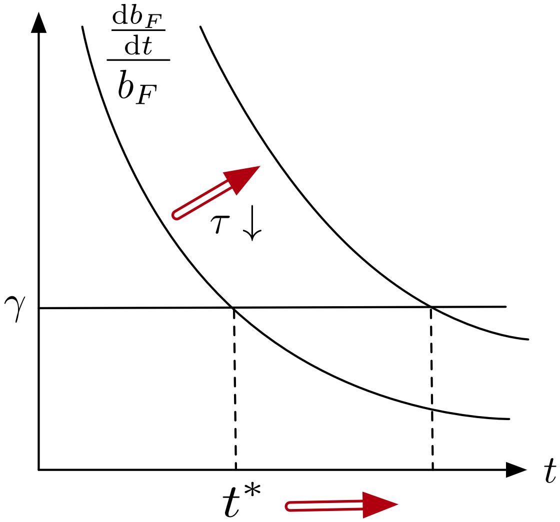
$$\frac{\frac{\mathrm{d}b_F}{\mathrm{d}t}(t)}{b_F(t)} = \gamma$$

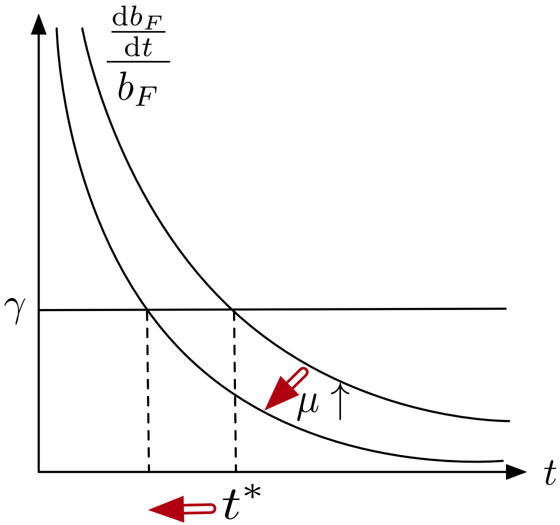
or

$$\frac{bp\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\left(\frac{1}{\tau+tp}\right)^{\frac{3}{2}}}{a\mu - b\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\left(\frac{1}{\tau+tp}\right)^{\frac{1}{2}}} = \gamma$$



$\frac{\frac{\mathrm{d}b_F}{\mathrm{d}t}}{b_F}$
γ
t
t^*


$\frac{\frac{\mathrm{d}b_F}{\mathrm{d}t}}{b_F}$
$\tau \downarrow$
γ
t
t^*


$\frac{\frac{\mathrm{d}b_F}{\mathrm{d}t}}{b_F}$
γ
$\mu \uparrow$
t
t^*

Proposition

Assume the discounting cost model and $a\mu - b\left(\frac{2}{\pi}\right)^{\frac{1}{2}} > 0$:

1. An increase in the variance of the relevant stochastic variable decreases the quantity of current investment:

$$\tau \downarrow \Longrightarrow t^* \uparrow$$

2. An increase in the mean of the relevant stochastic variable increases the quantity of current investment:

$$\mu \uparrow \Longrightarrow t^* \downarrow$$

- If the mean μ of the prior increases, the investment becomes more profitable on average. Thus, the agent will quit information acquisition earlier

Summary

	Preference over menus	Stochastic choice
Additive costs	de Oliveira, Denti, Mihm, and Ozbek (2017)	Caplin and Dean (2015)
Discounting costs	This paper	Chambers, Liu, and Rehbeck (2018)
General costs		Chambers, Liu, and Rehbeck (2018)