Farsighted Stable Sets of Tariff Games*

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April 17, 2012

Abstract

This article analyzes the tariff negotiation game between two countries when the countries are sufficiently farsighted. It extends the research of Nakanishi (2000) and Oladi (2005) for the tariff retaliation game in which countries take into account subsequence retaliations that may occur after their own retaliation. We show that when countries are sufficiently farsighted, all farsighted stable sets of the tariff game are singletons, which are Pareto efficient and strictly individually rational tariff profiles. These results hold regardless of whether coalitional deviations are allowed or not.

*The authors thank Keisuke Bando and Shin Kishimoto for helpful comments and suggestions.

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1 Introduction

This article analyzes the tariff negotiation game between two countries when the countries are sufficiently farsighted. Primary papers in the literature, such as Johnson (1953–1954) envision a scenario in which countries choose an optimal tariff rate given that the other country does not change its tariff rate. Tower (1975) and Rodriguez (1974) have carried this analysis over to the game in which countries, instead of choosing the tariff rates, choose export or import quotas. Although not explicit in their formulation, their framework employs an equilibrium concept similar to that of Nash equilibrium. In these models, each country successively chooses a tariff rate or a quota level under the assumption that the other country stays put.

However, when each country chooses such an optimal level, it does not take into account the consequences of such actions that it triggers, including the possibility that the other country may retaliate in response. Recently, Nakanishi (1999), for the quota game, and Oladi (2005) and Nakanishi (2000), for the tariff game, have applied the theory of social situations of Greenberg (1990) to the export quota game and the tariff game respectively to capture this possibility in their model. However, the domination relation that their findings are based on does not take into account the situation in which players are not myopic. In this paper, we analyze the stable outcomes in tariff games when players can sufficiently take into account the consequences of their deviations and are only interested in the final outcomes as results of such deviations. To do so, we apply the farsighted stable set to tariff games.

There has been a growing literature of the application of farsighted stable set of Chwe (1994). The starting point of the argument for the farsighted stable set start with the argument by Harsanyi (1974) and Chwe (1994) that the classic stable set of von Neumann and Morgenstern (1953) uses a domination relation that is myopic. Attempting to take into account sequences of deviations that may occur, Harsanyi (1974) and Chwe (1994) define a domination relation, called indirect domination, which is then used to define the farsighted stable set. This solution concept has been used in papers, including, to the authors' best knowledge, Suzuki and Muto (2005) and Kamijo and Muto (2010) in sending a message that farsightedness is the key element in reaching Pareto efficient outcomes. This message has to be taken cautiously since they allow coalitional deviations – that is, simultaneous deviation by multiple players. The juxtaposition of the results in Suzuki and Muto (2000), Masuda (2002), Nakanishi (2009), and Kawasaki and Muto (2009) reveal that there is not a direct relationship between the efficiency of the results in farsighted stable sets and the rules of the game ascertaining the allowance of coalitional deviations.

In light of the aforementioned papers in the literature, we analyze the farsighted stable sets of two different games of tariff games. In the first model, we allow for coalitional deviations – simultaneous deviations made by both countries. The first model corresponds to the rule of negotiating outlined in Oladi (2005) and the first model in Nakanishi (2000), both of which consider tariff retaliation games. In the second model, we disallow coalitional deviations. This restriction can be interpreted as an alternating negotiation game in which one player proposes one tariff, while in the next step, the other player can respond. This model is closely related to Nakanishi (1999), which also restricts deviations to those made by individual players in a quota retaliation game.

We show that in both games, the tariff choices by two countries that is Pareto efficient and strictly individually rational constitutes a singleton farsighted stable set. Moreover, we can show that no other farsighted stable sets exist in these two games. Thus, the rules of the game regarding coalitional deviations do not affect the outcome of the results, although the proof of the statement is far more involved in the second game. Unlike Nakanishi (2000), to achieve efficiency the only main addition to the original model in Oladi (2005) that is used is that countries are sufficiently farsighted, and in addition, outcomes that are not individually rational are not supported by a farsighted stable set.

One possible criticism to this approach is that it requires the players to be able to foresee events multiple steps ahead. However, as will be apparent in the proofs of the statements of this paper, we do not need to assume a substantial amount of farsightedness to establish the results. All of the results hold when player can foresee at least four steps ahead.

Our main focus of this paper is on tariff games, but we can easily use the same logic employed in this paper to show a similar result for the export quota game.

The rest of the paper proceeds as follows. In the next section, we introduce two models of the tariff game as mentioned in the introduction. In section 3, we review the literature on farsighted stable sets and provide key definitions and their properties. In sections 4 and 5, we present the results for the two models. We conclude in section 6.

2 Two Models of the Tariff Game

In this section, we introduce the tariff game. The main components of the game follow those in Nakanishi (2000). First, we introduce some basic notations. Let $G = (N, (X_i)_{i \in N}, (U_i)_{i \in N})$ be a game in strategic form where X_i is the set of strategies for player $i \in N$. In the tariff game, X_i is the set of tariffs from which a country can choose from. U_i is the payoff function for player i.

To incorporate farsightedness into this framework, Chwe (1994) defines the effectiveness relation $\rightarrow_S$ for each $S \subseteq N$ as a binary relation on $X = \prod_{i \in N} X_i$ such that $x \rightarrow_S y$ denotes that players in S can realize the outcome y when x is the status quo. A concrete definition of $\rightarrow_S$ depends on the context of how the game is defined – including, for example, whether coalitional deviations are allowed or not.[1] The difference of the two models considered in this paper come from how this relation is defined.

In the first model, we do not impose any additional restrictions to the ones that are implied by a game in strategic form. Therefore, in the first model, we have

$$x \rightarrow_S y \Leftrightarrow x_i = y_i \ \forall i \in N \setminus S,$$

where the latter condition is dropped if $S = N$. This model closely resembles the first model considered in Nakanishi (2000) and Oladi (2005). In their model, they consider the direct domination relation, one in which indirect domination holds where the length of the deviation is one. Oladi (2005) shows that the set of Pareto efficient tariffs is a stable set defined by direct domination, but Nakanishi (2000) shows the existence of several others and is able to eliminate these other solutions by restricting the deviation relation by what he calls the WTO tariff concession rules.

In the second model, we consider a situation resembling closely to an alternating offer model. In this situation

$$x \rightarrow_S y \Leftrightarrow |S| = 1 \text{ and } x_i = y_i \ \forall i \in N \setminus S.$$

The above condition states that only one player can deviate at a time. Nakanishi (1999) imposes this condition on the quota retaliation game. To reflect the alternating part, we define an indirect domination relation with the effectiveness relation in this situation.

We now explain the main components of the tariff games themselves. These components pertain to both models; therefore, all the observations and properties introduced here hold for the first model as well as the second model.

Let $N = \{1, 2\}$ be the set of players, where in the tariff games, each country is a player. Throughout these two terms will be used interchangeably.

X_i represents the set of tariffs that country i can choose and is defined as $X_i = (-1, \bar{t}_i]$ where $\bar{t}_i$ represents the highest tariff rate that is permitted. A negative tariff rate is defined to be a subsidy from one country to the other; the value -1 is not included

[1]The effectiveness relation is a simplified form of the inducement correspondence in the theory of social situations. See Greenberg (1990) for details.

in the set, since the prices would be undefined. Although a negative tariff rate seems impractical, we include the possibility here to make direct comparisons to Oladi (2005) and Nakanishi (2000) both of which include them as well. All of our results hold (much more easily) if we restrict our attention to only nonnegative tariff rates. $X = X_1 \times X_2$ represents the set of possible outcomes resulting from the choices of countries 1 and 2. Throughout this paper we call an element $t \in X$ a tariff profile or simply an outcome.

Following Nakanishi (2000), define X^o as the set of tariff profiles at which there is no trade, because either country or both have set a relatively high tariff rate, thereby discouraging trade. We assume that X^o contains tariff profiles x such that $x_i = \bar{t}_i$ and $x_j > 0$. We let $U_i(x) = \bar{u}_i$ for all outcomes $x \in X^o$. Let $X^* = X \setminus X^o$ be the set of tariff profiles at which there is a positive amount of trade.

We assume that the utility functions defined on the tariff profiles are continuous on X; for each fixed x_j, U_i is quasiconcave in x_i; and for each fixed x_i, U_i is decreasing in x_j along X^*.[2]

A tariff profile x is said to be *Pareto dominated* by another tariff profile y if $U_i(x) \leq U_i(y)$ for all i and $U_j(x) < U_j(y)$ for some j. In that instance, we also say that y Pareto dominates x, and we denote this by yPx. A tariff profile x is *Pareto efficient* if there does not exist y such that yPx.

From Mayer (1981) and Nakanishi (2000), the set of Pareto efficient tariff combinations, denoted by E, is given by the union of the following sets.

$$\begin{aligned}
E^* &= \{x \in X : (1 + x_1)(1 + x_2) = 1\} \\
E_1 &= \{x \in X : x_1 = e_1^1, x_2 \leq e_2^1\} \\
E_2 &= \{x \in X : x_1 \leq e_1^2, x_2 = e_2^2\}
\end{aligned}$$

where $e^i = (e_1^i, e_2^i)$ is such that $e_i^i = \bar{t}_i$ and e_j^i is such that $(1 + \bar{t}_i)(1 + e_j^i) = 1$.

Also, for each $x = (x_1, x_2)$, define $L(x) = \{y \in X : y_1 \leq x_1, y_2 \leq x_2\}$ and $H(x) = \{y \in X : y_1 \geq x_1, y_2 \geq x_2\}$ and for each subset $A \subset X$, $L(A) = \bigcup_{x \in A} L(x)$ and $H(A) = \bigcup_{x \in A} H(x)$.

In addition, we assume the following on the indifference curves of the two countries in the tariff space:

(A1) For $x, y \in H(E^*) \cap X^*$ such that $U_i(x) = U_i(y)$, if $x_i < y_i$, then $U_j(x) > U_j(y)$ $(j \neq i)$

[2] For a detailed explanation of the assumptions of the utility functions and derivations from primitives such as exports and imports, see Mayer (1981), Oladi (2005), and Nakanishi (2000).

(A2) For $x, y \in L(E^*)$ such that $U_i(x) = U_i(y)$, if $x_i < y_i$, then $U_j(x) < U_j(y)$ $(j \neq i)$.

The two conditions combined imply that i's indifference curve and j's indifference curve can cross at most twice. If they do cross twice, they cross once in $L(E^*)$ and once in $H(E^*) \cap X^*$. These conditions are used mostly for Lemma 6.

Denote by m_i, i's maximin value of the game. That is, for $j \neq i$,

$$m_i = \max_{x_i \in X_i} \min_{x_j \in X_j} U_i(x_i, x_j).$$

Let X_i^M denote set of $x_i \in X_i$ that solve the above problem. Each element in X_i^M is the maximinimzer for player i. Based on the assumptions of the tariff game, we can show that the maximin value of the game is $\bar{u}_i$ for player i and that $X_i^M = [0, \bar{t}_i]$.

Let X^{SI} be the set of *strictly individually rational* strategy profiles, which is the set of $x \in X$ such that for each i, $u_i(x) > m_i$ for each i. In comparison, a strategy profile is said to be *individually rational* if the previous inequality holds with a weak inequality for each i. Thus, the set of strictly individually rational tariff combinations is given by

$$X^{SI} = \{x \in X : U_i(x_1, x_2) > \bar{u}_i \ \forall i \in N\}.$$

We now introduce the reaction function $\phi_i(x_j)$ for each i and $j \neq i$ defined in Nakanishi (2000). $\phi_i(x_j)$ assigns a tariff rate x_i^* such that

$$U_i(x_i^*, x_j) \geq U_i(x_i, x_j) \ \forall x_i \in X_i.$$

For $x_j \neq \bar{t}_j$, the x_i^* that satisfies the above condition is unique. However, when $x_j = \bar{t}_j$, for any $x_i \geq 0$, $U_i(x_i, \bar{t}_j) = \bar{u}_i$, and for all $x_i < 0$, $U_i(x_i, \bar{t}_j) < \bar{u}_i$. By convention, we set $\phi_i(\bar{t}_j) = 0$. Furthermore, we assume that ϕ_i is a decreasing function of x_j and $\phi_i(0) < \bar{t}_i$. These assumptions are the same as those made in Nakanishi (2000). From these assumptions, we can deduce that these reaction functions cross at a point x^* such that $x_i^* > 0$ for both players, implying that there exists a Nash equilibrium in which both countries set a positive tariff rate. Note that for such x^*, $x^* \notin E$, and $U_i(x^*) > \bar{u}_i$ for all i.

Nakanishi (2000) then shows the following property of ϕ_i. This property is used mostly in proving Lemma 6.

Proposition 1. *Fix $\bar{x}_j$ and let ϕ_i be i's best reply function. Then, the following properties of ϕ_i hold.*

1. *If $x_i < y_i \leq \phi_i(\bar{x}_j)$, then $U_i(x_i, \bar{x}_j) < U_i(y_i, \bar{x}_j)$.*

2. If $x_i > y_i \geq \phi_i(\bar{x}_j)$ and $(x_i, \bar{x}_j), (y_i, \bar{x}_j) \in X^*$, then $U_i(x_i, \bar{x}_j) < U_i(y_i, \bar{x}_j)$.

For a graphical depiction of the tariff game – such as indifference curves, reaction curves, and the Pareto efficient region E^* – see Nakanishi (2000) and Oladi (2005).

We consider two different models of the above tariff negotiation game, where the difference will be reflected in how the effectiveness relation $\rightarrow_S$ is defined for each $S \subseteq N$. The reason for doing so is to provide a unifying analysis based on the different rules of deviation imposed by Nakanishi (1999) and by Oladi (2005). In attempts to capture retaliation more realistically, these two papers formulate their games on some contingent threats situation as defined by Greenberg (1990). Nakanishi (1999) formulates the quota retaliation game as an individual contingent threats situation of Greenberg (1990) that does not permit coalitional deviations, while Oladi (2005) uses the coalitional contingent threats situation of Greenberg (1990) which allows coalitional deviations. In their models, how the effectiveness relation is defined plays a key role in their results.

Their results can be summarized in the following way. Define x to be directly dominated by y if there exists $S \subseteq N$ such that $x \rightarrow_S y$ and $U_i(x) < U_i(y)$ for all $i \in S$. That is, x is directly dominated by y if there exists a coalition S that can enforce y from x and benefit from doing so. Using this direct domination relation, one can define the stable set, first defined in von Neumann and Morgenstern (1953), of this game, along the lines as outlined in Greenberg (1990).

Nakanishi (2000) shows that under this setup there exist multiple stable sets, most of which involve inefficient outcomes. By imposing restrictions on the possible deviations to emulate the situation as prescribed by the WTO rules, he shows that the set E is the unique stable set, restoring efficiency. On the other hand, E may include outcomes that are not individually rational.

As Harsanyi (1974) and Chwe (1994) note, the direct domination relation is myopic in that players do not take into account subsequent deviations that may occur after the initial deviation. We wish to show that by using the domination relation that incorporates farsightedness of the players, that efficient outcomes that are strictly individually rational are stable, and those outcomes are the only ones that can be stable. In the next section, we explain the indirect domination relation built off of the two situations introduced in this section.

3 Farsighted Stable Sets in Strategic Form Games

A strategy profile x is said to be *indirectly dominated* by another strategy profile y, denoted by $x \ll y$ if there is a sequence of strategy profiles $a^0, a^1, \cdots, a^m$ and coalitions (subsets of N) $S_0, S_1, \cdots, S_{m-1}$ with $a^0 = x$ and $a^m = y$ such that for each $k = 0, 1, \cdots, m-1$,

- $a^k \rightarrow_{S_k} a^{k+1}$ and
- $U_i(a^k) < U_i(y)$ for all $i \in S_k$.

The first condition states that y can be reached from x through a sequence of deviations. The second condition states that each individual in each deviating coalition is made better off in the final outcome y.

This definition of indirect domination can be applied to both models as is written above. For example, in the first model, the definition accommodates deviations made by two or more players at the same time, while in the second model, the first of the two conditions above imply that only individual deviations are accounted for. However, to be clear, we provide a definition and a separate notation for the second model.

Formally, a strategy profile x is said to be *indirectly dominated through individual deviations* by another profile y, denoted by $x \ll_I y$ if there exist sequences of strategy profiles $a^0, a^1, \cdots, a^m$ and subsets of N denoted by $i_0, i_1, \cdots, i_{m-1}$ with $a^0 = x$ and $a^m = y$ such that for each $k = 0, 1, \cdots, m-1$,

- $i_k \neq i_{k+1}$
- $a^k \rightarrow_{i_{k+1}} a^{k+1}$
- $U_{i_k}(a^k) < U_{i_k}(y)$.

The first condition rules out consecutive deviations made by the same player. This condition is not restrictive, since even if we were to allow such consecutive deviations, we can surpress the sequence by making them into one single deviation.

This second definition is specifically defined to make a direct analogy to papers in the literature that use noncooperative frameworks. In general, this restriction can lead to a different set of results as can be seen by comparing the results of Suzuki and Muto (2000) for the two-player prisoners' dilemma game and by comparing the results of Suzuki and Muto (2005) and Nakanishi (2009) for prisoners' dilemma games.

A *farsighted stable set* V with respect to the domination relation $\ll$ is a subset of X that satisfies the following two conditions.

- For every $x, y \in V$, $x \ll y$ does not hold.
- For every $x \notin V$, there exists $y \in V$ such that $x \ll y$.

Similarly, we can define a farsighted stable set with respect to $\ll_I$ by replacing each $\ll$ with $\ll_I$ in the above definition.

As a general rule, Shino and Kawasaki (2012) show that farsighted stable sets of strategic form games only contain individually rational strategy profiles. This property is not shared with its myopic counterpart, as can be seen by the results of Nakanishi (2000) for the tariff game. In fact, the results using the myopic stable set support tariff choices in which one country subsidizes a substantial amount. Such a pair of tariffs is Pareto optimal because one country benefits greatly from the subsidy of the other country, while that country is hurt by it, especially if the other country is not subsidizing.

There is another property of the indirect domination relation pertaining to the idea of the maximin strategy. The result is first introduced in Suzuki (2002), and its proof follows from the definition of the maximin strategy and the definition of indirect domination. For completeness, we provide the proof here.

Lemma 1. *Take any x such that $u_1(x) \leq m_1$ where m_1 is the maximin value for player 1 and let x_1^M be player 1's maximin strategy and x_2^M be player 2's maximin strategy and suppose $x_1^M \neq x_1$. Then, neither $x^M = (x_1^M, x_2^M) \ll x$ nor $x^M \ll_I x$ can hold. A similar result holds for player 2.*

Proof. We show the proof for the case of the indirect domination relation $\ll$, as the proof for $\ll_I$ holds by the same argument. Let x satisfy the conditions outlined in the statement, and suppose by way of contradiction that $x^M \ll x$ were to hold. By definition, there exists a sequence of coalitions $S_0, S_1, \cdots, S_{K-1}$ and $x^M = a^0, a^1, \cdots, a^K = x$ and outcomes such that the definition of $\ll$ is satisfied. Consider the first coalition in the sequence, S^0. Because $U_1(x) \leq m_1 \leq U_1(x^M)$ hold by definition of the maximin strategy and how x is defined, $S^0 = \{2\}$ – that is, S_0 cannot include player 1. Consider x^1 and S_1. Because $x_1^1 = x_1^M$, we still have $U_1(x) \leq m_1 \leq U_1(x^1)$, which implies by definition of $\ll$ that $S_1 = \{2\}$. By continuing this argument, we arrive that player 1 is not included in any of the coalitions $S_0, \cdots, S_{K-1}$. On the other hand, because $x_1^M \neq x_1$ and by definition of $\rightarrow$, there must exist some k such that S_k includes player 1, which is a contradiction. □

4 When Coalitional Deviations are Allowed

Lemma 2. *Let $x \in X$ and $y \in X^{SI}$ such that for some $i \in \{1, 2\}$, $U_i(x) < U_i(y)$. Then, $x \ll y$.*

Proof. Let i be such that $U_i(x) < U_i(y)$ and $y \in X^{SI}$. Then, $x \ll y$ via the sequence

$$x \to_i (\bar{t}_i, x_j) \to_j (\bar{t}_i, \bar{t}_j) \to_N y.$$

To check that $x \ll y$ indeed holds, note that $U_j(\bar{t}_i, x_j) \leq \bar{u}_j < U_j(y)$, and the last step follows from the fact that $y \in X^{SI}$ and $U_k(\bar{t}_i, \bar{t}_j) = \bar{u}_k$ for each $k = i, j$. □

The following corollary shows that any x that is Pareto efficient and strictly individually rational is a singleton farsighted stable set and immediately follows from the previous lemma. Because $X^{SI} \cap E \neq \emptyset$, the corollary also establishes the existence of a farsighted stable set.

Corollary 1. *For any $x \in X^{SI} \cap E$, $\{x\}$ is a farsighted stable set.*

Proof. Take $x \in X^{SI} \cap E$. Internal stability of $\{x\}$ is trivial since the set is a singleton, so to show that this set satisfies external stability, take any $x' \neq x$. Because $x \in E$, there exists $i \in N$ such that $U_i(x') < U_i(x)$. By Lemma 2 and the fact that $x \in X^{SI}$, $x' \ll x$. □

The next lemma shows that there cannot exist a farsighted stable set that includes strictly individually rational outcomes that are not Pareto efficient.

Lemma 3. *For any $x \notin E$ and $x \in X^{SI}$, there cannot exist a farsighted stable set V such that $x \in V$.*

Proof. Suppose that there exists a farsighted stable set V that includes such x. Because $x \notin E$, there exists y such that yPx, which implies that there exists some $i \in N$ such that $U_i(x) < U_i(y)$. Moreover, since $\bar{u}_j < U_j(x) \leq U_j(y)$ for all $j \in N$, we have that $y \in X^{SI}$. Thus, by Lemma 2, $x \ll y$ holds. Internal stability of V implies that $y \notin V$. By external stability of V, there exists $z \in V$ such that $y \ll z$. By definition of $\ll$, $y \ll x$ cannot hold, which then implies that $x \neq z$. By definition of $\ll$ and the fact that yPx, $U_j(x) \leq U_j(y) < U_j(z)$ holds for some $j \in N$. If $z \in X^{SI}$, then Lemma 2 yields $x \ll z$, contradicting the internal stability of V. If not, then for some k, $U_k(z) = \bar{u}_k < U_k(x)$. By Lemma 2, $z \ll x$, contradicting the internal stability of V. Therefore, no such V can exist. □

Theorem 1. *All farsighted stable sets V are of the form $\{x\}$ where $x \in X^{SI} \cap E$.*

Proof. By Lemma 3 and Corollary 1, if there were a farsighted stable set V different from above, then V must contain an outcome y such that $y \notin X^{SI}$. However, y must be individually rational (Lemma 2.1 of Shino and Kawasaki (2012)), implying that $U_i(y) = \bar{u}_i$ for some i and $U_j(y) \geq \bar{u}_j$. Let $i = 1$ for simplicity of argument. By the assumptions made on the utility functions, there exists a Nash equilibrium $x^* = (x_1^*, x_2^*)$ of the tariff game and $x^* \notin E$. Moreover, $x_1^* > 0$ and $x_2^* > 0$ hold, and each strategy is a maximin strategy for players 1 and 2 respectively. Because ϕ_i is single-valued when $x_j > 0$, $U_i(x_i^*, x_j^*) > U_i(\bar{t}_i, x_j^*) = \bar{u}_i$ for all i. This fact establishes $(x_1^*, x_2^*) \in X^{SI}$, which implies by Lemma 3 that $(x_1^*, x_2^*) \notin V$. On the other hand, there cannot exist $x \in V$ such that $x^* \ll x$ for the following reasons:

- If $x_1 \neq x_1^*$, then $U_1(x) = \bar{u}_1$ implies that $x^* \ll x$ cannot hold. (Lemma 1)
- If $x_1 = x_1^*$, then $x_2^* \neq x_2$. However, because x^* is a Nash equilibrium, $U_2(x) \leq U_2(x^*)$. We also have $U_1(x) = \bar{u}_1 < U_1(x^*)$, so neither player can deviate in the first step of $\ll$.

Therefore, V does not satisfy external stability, which is a contradiction. □

5 When only Individual Deviations are Allowed

In this section, we show that for the negotiation game of alternating offers, we can obtain a parallel result – every farsighted stable set is a singleton comprised of a tariff combination that is Pareto efficient and strictly individually rational. Although, the result itself is the same as before, the proof of this statement for this particular model is much more involved, since the two indirect domination relations $\ll$ and $\ll_I$ are not equivalent.

To prove our preliminary results, we need to introduce the following subsets of X^*:

- $X^- = \{x \in X : x_i < 0 \text{ and } U_i(x) \leq U_i(\bar{t}_i, x_j) \text{ for all } i = 1, 2; j \neq i\}$
- $X^+ = X^* \setminus X^- = \{x \in X^* : x_i \geq 0 \text{ for some } i\} \cup \{x \in X^* : U_i(x) > U_i(\bar{t}_i, x_j) \text{ for some } i\}$
- $X^{--} = \{x \in X : x_i < 0 \text{ for all } i\}$
- $X^{++} = X^* \setminus X^{--} = \{x \in X^* : x_i \geq 0 \text{ for some } i\}$

That is, X^- is the set of tariff profiles at which both countries subsidize each other and with the additional property that for every $x \in X^-$ and every i and $y_i \in X_i$ with $y_i \leq x_i$, $U_i(x_i, x_j) \geq U_i(y_i, x_j)$, which follows from Proposition 1. $X^+ = X^* \setminus X^-$ is the complement of X^- and is the set of tariff profiles where the "usual" results hold (see Lemma 4). X^{--} is the set of tariff rates in which both countries are subsidizing, while

X^{++} denotes one in which at least one imposes a nonnegative tariff rate. Note that, by how these sets are defined, $X^{++} \subset X^{+}$ and $X^{-} \subset X^{--}$.

For this game, the analogue of Lemma 2 may not hold over all of X^*. To see this, recall that in the proof of Lemma 2, the last step involved a simultaneous deviation by both players. It is then not trivial to break this step into two separate steps and still satisfy the conditions necessary in establishing indirect dominance. The next two lemmas show that the result still holds for X^+, while the same may not hold for X^-.

Lemma 4. *For any $x \in X$ and $y \in X^{SI} \cap X^+$ such that $U_i(x) < U_i(y)$ for some i, $x \ll_I y$.*

Proof. Without loss of generality let country 1 be such that $U_1(\bar{t}_1, y_2) < U_1(y)$. Suppose country 2 is such that $U_2(x) < U_2(y)$. Then, consider the following sequence:

$$x = (x_1, x_2) \rightarrow_2 (x_1, \bar{t}_2) \rightarrow_1 (y_1, \bar{t}_2) \rightarrow_2 (y_1, y_2) = y.$$

The above sequence establishes $x \ll_I y$ because of the following inequalities:

- $U_1(x_1, \bar{t}_2) = \bar{u}_1 < U_1(y)$ by the fact that $y \in X^{SI}$,
- $U_2(y_1, \bar{t}_2) < U_2(y)$ by definition.

If country 1 is such that $U_1(x) < U_1(y)$, consider the following sequence:

$$x = (x_1, x_2) \rightarrow_1 (\bar{t}_1, x_2) \rightarrow_2 (\bar{t}_1, \bar{t}_2) \rightarrow_1 (y_1, \bar{t}_2) \rightarrow_2 (y_1, y_2) = y.$$

By the same logic as above, $x \ll_I y$ holds under this case as well. □

The following result is the analogue to Corollary 1 and is immediate from the previous lemma.

Corollary 2. *For any $x \in X^{SI} \cap E$, $\{x\}$ is a farsighted stable set with respect to $\ll_I$.*

Proof. Internal stability is trivially satisfied because the set is a singleton. To show external stability, take any $y \neq x$. Because $E \subset X^+$ and from the fact that $x \in E$, there exists $i \in N$ such that $U_i(y) < U_i(x)$. Then, by Lemma 4, $y \ll_I x$. □

Next, we wish to derive a result parallel to Lemma 3. To do so, we need to show that the region X^- does not interfere with the logic used in the previous game. The following lemma shows that tariff profiles in X^- cannot indirectly dominate those in X^{++}. At the same time, the lemma illustrates how Lemma 2 may not hold for this model.

Lemma 5. *For any $y \in X^-$, if $x \ll_I y$ holds, then $x \in X^{--}$. Equivalently, for any $y \in X^-$ and $x \in X^{++}$, $x \ll_I y$ cannot hold.*

Proof. Suppose $x \ll_I y$ holds with $y \in X^-$. Then, there exists a sequence of players $i_0, i_1, \cdots, i_r$ and sequence of tariff profiles $x^0, \cdots, x^{K+1}$ with $x^0 = x$ and $x^{K+1} = y$ such that $x^k \to_{i_k} x^{k+1}$ for all $k = 0, 1, \cdots, K$ and $U_{i_k}(x^k) < U_{i_k}(y)$ for all $k = 0, 1, \cdots, K$. Without loss of generality, we can consider a simple sequence in which the sequence of players is alternating such that $i_k \neq i_{k+1}$ for all k and $i_K = 1$. We claim that $x_1^k < y_1$ for each $k = 0, 1, \cdots, K$ and $x_2^k < y_2$ for each $k = 0, 1, \cdots, K-1$,[3] which then implies $x_1 = x_1^0 < y_1 < 0$ and $x_2 = x_2^0 < y_2 < 0$. To show these inequalities, we also claim that for each $k = 0, 1, \cdots, K$ and $i = 1, 2$, $U_i(y) \leq U_i(\bar{t}_i, x_j^k)$ where $j \neq i$.

The proof is by backwards induction. Consider $k = K$, the last step in the sequence. Without loss of generality, let $i_K = 1$. Then, $x^K = (x_1^K, y_2)$ by definition of $\to_1$. From $y \in X^-$, we know that $y_2 < 0$. By definition of $\ll_I$, we must also have $U_1(x_1^K, y_2) < U_1(y_1, y_2)$. We first show that $x_1^K < y_1$ holds. If $x_1^K \geq y_1$, we must have

$$U_1(x_1^K, y_2) \geq \min\{U_1(\bar{t}_1, y_2), U_1(y_1, y_2)\} \geq U_1(y),$$

where the first inequality follows from the quasiconcavity of U_1, and the second inequality follows from $y \in X^-$. The above inequalities contradict the definition of $\ll_I$. Therefore, $x_1^K < y_1 < 0$.

To show the last part, suppose that $U_1(x^K) > U_i(\bar{t}_1, x_2^K)$. Then, the following inequality holds by the fact that $y \in X^-$ and that $x^K \to_1 y$:

$$U_1(y) > U_1(\bar{t}_1, x_2^K) = U_1(\bar{t}_1, y_2) \geq U_1(y),$$

which then contradicts the second condition in the definition of $\ll_I$. Therefore, we must have $U_1(y) \leq U_1(\bar{t}_1, x_2^K)$. Similarly, if $U_2(y) > U_2(x_1^K, \bar{t}_2)$, then the following inequalities hold, which lead to a contradiction:

$$U_2(y) > U_2(x_1^K, \bar{t}_2) > U_2(y_1, \bar{t}_2) \geq U_2(y).$$

Therefore, we must also have $U_2(y) \leq U_2(x_1^K, \bar{t}_2)$.

Now, suppose that $x_1^k < y_1$ and $x_2^k < y_2$ and $U_i(y) \leq U_i(\bar{t}_i, x_j^k)$ holds for each $i = 1, 2$ and $j \neq i$. We first show that $x_1^{k-1} < y_1$ and $x_2^{k-1} < y_2$. Without loss of generality, suppose that $i_{k-1} = 2$. Then, $x_1^{k-1} = x_1^k < y_1$ holds immediately. If $x_2^{k-1} \geq y_2$, then we

[3]This inequality is not considered when $K = 0$.

have the following inequalities:

$$U_2(x^{k-1}) = U_2(x_1^k, x_2^{k-1}) \geq \min\{U_2(x_1^k, \bar{t}_2), U_2(x_1^k, y_2)\} \geq U_2(y).$$

The first inequality holds since U_2 is quasiconcave in its own argument. The second inequality holds from the assumption that $y \in X^-$ (which implies $U_2(x_1^k, \bar{t}_2) \geq U_2(y)$) and that $x_1^k < y_1$ from the induction hypothesis and U_2 being decreasing in the tariff rate of country 1 imply $U_2(x_1^k, y_2) \geq U_2(y)$. On the other hand, the definition of $\ll_I$ implies that $U_2(x^{k-1}) < U_2(y)$, which leads to a contradiction. Therefore, we must have $x_2^{k-1} < y_2$.

To complete the induction argument, we need to show that $U_i(y) \leq U_i(\bar{t}_i, x_j^{k-1})$ for each i. First, consider $i = 1$ and suppose that $U_1(y) > U_1(\bar{t}_1, x_2^{k-1})$. Then, we have the following contradictory set of inequalities

$$U_1(y) > U_1(\bar{t}_1, x_2^{k-1}) > U_1(\bar{t}_1, y_2) \geq U_1(y),$$

where the second inequality follows from the fact that U_1 is decreasing in country 2's tariff and that $x^{k-1} < y_2$ holds from the previous part. Thus, $U_1(y) \leq U_1(\bar{t}_1, x_2^{k-1})$. Similarly, for $i = 2$,

$$U_2(y) > U_2(x_1^{k-1}, \bar{t}_2) > U_2(y_1, \bar{t}_2) \geq U_2(y).$$

A symmetric argument can be used in the case that $i_{k-1} = 1$. Therefore, we have shown that the claim holds. □

Recall that the goal is to derive a result concerning the exclusion of Pareto inferior and strictly individually rational outcomes from any farsighted stable sets. The next result shows that we can take a tariff profile from the set X^{++} that Pareto dominates the inefficient ones without any loss of generality.

Lemma 6. *For any $x \notin E$, there exists $y \in X^{++}$ such that yPx.*[4]

Proof. Take $x \notin E$. Then, there exists $y \in X$ such that yPx. If $y \in X^{++}$, we are done. If $y \in X^{--}$ but y is such that $U_i(y) \leq \bar{u}_i$ for each i, then we can then take $z \in X^{SI} \cap X^*(\subset X^{SI} \cap X^{++})$ such that zPy, which in turn implies zPx. Thus, consider the case in which $y \in X^{--}$. Because $y \in X^{--}$, and $E \subset X^{++}$, $y \in L(E^*)$. Consider an

[4] According to the proof of the main theorem of Oladi (2005), each Pareto inefficient outcome is Pareto dominated by some Pareto efficient tariff combination. However, this fact is stated without a proof in Oladi (2005) and is claimed to follow from the definition of Pareto efficiency. For completeness, we provide a proof of a weaker statement here, since this version suffices for the proof of the results that follow.

indifference curve for player 1 through y. This indifference curve must cross E^* at some point. Label this intersection as z. We claim that zPy which implies that $z \in X^{++}$ and zPx hold.

First, note that $y_1 < z_1$. Otherwise, if $y_1 > z_1$,[5] we must then have $y_2 \leq z_2$. To see this, if $y_2 > z_2$, then

$$U_1(y_1, y_2) > U_1(z_1, y_2) \geq U_1(z_1, z_2).$$

The first inequality follows from Proposition 1, and the second inequality follows from the fact that U_1 is decreasing in player 2's tariffs. These inequalities imply that $U_1(y) \neq U_1(z)$, but y and z lie on the same indifference curve, which lead to a contradiction. Thus, $y_2 \leq z_2$.

Now, $y_1 > z_1$ and $y_2 \leq z_2$ imply the following inequalities

$$U_2(y_1, y_2) \leq U_2(y_1, z_2) < U_2(z_1, z_2),$$

where the first inequality follows from Proposition 1, and the second inequality follows from U_2 being a decreasing function of player 1's tariffs. On the other hand, assumption (A2) implies that $U_2(y) > U_2(z)$, which contradicts the above inequality. Thus, $y_1 < z_1$.

Now, by (A2), $U_2(y) < U_2(z)$. Thus, with $U_1(y) = U_1(z)$ and $U_2(y) < U_2(z)$, we can conclude that zPy. □

Now, we finally have the following analogue of Lemma 3.

Lemma 7. *For any $x \in X^{SI}$ with $x \notin E$, there does not exist a farsighted stable set V with respect to $\ll_I$ such that $x \in V$.*

Proof. First, we show that there cannot exist a farsighted stable set V that includes an x with $x \in X^{SI} \cap X^+$ and $x \notin E$. Suppose that there exists a farsighted stable set V such that $x \in V$. Because x is not Pareto efficient, there exists y such that $U_i(x) < U_i(y)$ for $i = 1, 2$. Also, we can find such a y with $y \in X^{++}$ by Lemma 6. Because $x \in X^{SI}$, $y \in X^{SI}$ must hold as well. Then, by Lemma 4, $x \ll_I y$, which in turn implies $y \notin V$ by internal stability. Thus, by external stability of V, there exists $z \in V$ such that $y \ll_I z$. Moreover, by definition of $\ll_I$, $y \ll_I x$ cannot hold, which implies that we must have $x \neq z$. By Lemma 5, $z \in X^+$. By definition of $\ll_I$, there exists some $j \in \{1, 2\}$ such that $U_j(x) < U_j(y) < U_j(z)$. If $z \in X^{SI}$, then $x \ll_I z$. If $z \notin X^{SI}$, then because $x \in X^{SI}$, there exists some j such that $U_j(z) = \bar{u}_j < U_j(x)$. By Lemma 4, since $x \in X^+ \cap X^{SI}$, $z \ll_I x$. In both cases, internal stability of V is violated. Therefore, there cannot exist a farsighted stable set V such that $x \in V$.

[5] $y_1 = z_1$ is ruled out since $z \in X^{++}$ and $y \in X^{--}$ and U_1 is strictly decreasing in player 2's tariffs.

To complete the proof, consider now $x \in X^{SI} \cap X^-$ and $x \notin E$. Then, by Lemma 6, there exists $y \in X^{++}$ that Pareto dominates x. By the same logic as above, $y \notin V$, which implies that by external stability, there exists some $z \in V$ such that $y \ll_I z$. However, by Lemma 5, $z \in X^+$. If $z \in X^{SI}$, then $x \ll_I z$, which contradicts the internal stability of V. If $z \notin X^{SI}$, by the fact that V can only include individually rational outcomes, for some i, we must have $U_i(z) = \bar{u}_i$. However, since $x \in X^{SI}$, $U_i(z) < U_i(x)$. By the same logic as before, there exists j with $U_j(x) < U_j(z)$, Thus, $j \neq i$. Moreover, we have that since $z \in X^+$, $U_j(z_i, \bar{t}_j) < U_j(z)$ because $U_i(\bar{t}_i, z_j) \geq \bar{u}_i = U_i(z)$ holds. To see the latter inequality, if $z_j \geq 0$, then $U_i(\bar{t}_i, z_j) = \bar{u}_i$, which implies in turn that if $z_j < 0$, then $U_i(\bar{t}_i, z_j) > U_i(\bar{t}_i, 0) = \bar{u}_i$. Then, $x \ll_I z$ through the following sequence, resulting in the violation of internal stability:

$$x = (x_i, x_j) \rightarrow_j (x_i, \bar{t}_j) \rightarrow_i (z_i, \bar{t}_j) \rightarrow_j (z_i, z_j) = z.$$

To check that $x \ll_I z$ indeed holds, we need to only check that $U_i(x_i, \bar{t}_j) < U_i(z)$ holds, but this fact can be checked by the fact that $U_i(x_i, \bar{t}_j) < U_i(0, \bar{t}_j) = \bar{u}_i = U_i(z)$ since $x_i < 0$. □

We can now state the main result of this section. The proof of the theorem follows from the same logic as Theorem 1 as Lemma 7 is the same as Lemma 3 and Corollary 2 is the same as Corollary 1.

Theorem 2. *If V is a farsighted stable set with respect to $\ll_I$, then $V = \{x\}$ for some $x \in X^{SI} \cap E$.*

6 Concluding Remarks

We have shown in this paper that farsightedness can lead to efficiency in tariff games regardless of how whether coalitional deviations are allowed or not. Moreover, we have made no additional assumptions, including those to the effectiveness relation, to the model of Nakanishi (2000) to obtain efficiency results. In addition, we have ruled out outcomes that are not individually rational. We can exclude outcomes in which one country is subsidizing too much, while under the basic WTO framework in Nakanishi (2000), this was not possible. We have seen how drastic the effect of the assumption of farsightedness alone has on the stability of the tariff choices of the countries.

While the main focus of this article was on tariff games, we can also consider the export quota retaliation game of Nakanishi (1999). It can be easily shown that the arguments used here can be applied almost in a straightforward manner to show a similar result.

That is, in the quota game, if both countries are sufficiently farsighted, all farsighted stable sets are singleton sets consisting of quota choices of both countries that are Pareto efficient, which in this game also imply strict individual rationality. The results hold regardless of whether coalitional deviations are allowed or not.

References

Chwe M S-Y (1994) Farsighted coalitional stability. J Econ Theory 63:299–325.

Greenberg J (1990) The theory of social situations: an alternative game-theoretic approach. Cambridge U Press, Cambridge.

Harsanyi J (1974) Interpretation of stable sets and a proposed alternative definition. Management Sci 20:1472–1495.

Johnson H G (1953–1954) Optimum tariffs and retaliation. Rev Econ Stud 21:142–153.

Kamijo Y, Muto S (2010) Farsighted stability of collusive price leadership. Japanese Econ Rev 61:455–465.

Kawasaki R, Muto S (2009) Farsighted stability in provision of perfectly "lumpy" goods. Math Soc Sci 58:98–109.

Masuda T (2002) Farsighted stability in average return games. Math Soc Sci 44:169–181.

Mayer W (1981) Theoretical consideration on negotiated tariff adjustment. Oxford Econ Papers 33:135–153.

Nakanishi N (1999) Reexamination of the international export quota game through the theory of social situations. Games Econ Behav 27: 132–152.

Nakanishi N (2000) The roles of rules on tariff concession in the WTO – a game-theoretical appraisal. Discussion Paper 0019, Graduate School of Economics, Kobe University.

Nakanishi N (2009) Noncooperative farsighted stable set in an n-player prisoners' dilemma. Int J Game Theory 38:249–261.

Oladi R (2005) Stable tariffs and retaliations. Rev Int Econ 13:205–215.

Rodriguez C A (1974) The non-equivalence of tariffs and quotas under retaliation. J Int Econ 4:295–298.

Shino J, Kawasaki R (2012) Farsighted stable sets in Hotelling's location games. Math Soc Sci 63:23–30.

Suzuki A (2002) Farsighted Stability in Strategic Form Games. Ph.D. thesis, Graduate School of Economics and Management, Tohoku University.

Suzuki A, Muto S (2000) Farsighted stability in prisoner's dilemma. J of the Operations Research Society of Japan 43:249–265.

Suzuki A, Muto S (2005) Farsighted stability in an n-person prisoner's dilemma. Int J Game Theory 33:431–445.

Tower E (1975) The optimum quota and tariff retaliation. Rev Econ Stud 42:623–630.

von Neumann J, Morgenstern O (1953) Theory of Games and Economic Behavior. Princeton University Press, Princeton, third edition.