Noncooperative Foundation of Nash Bargaining Solution under Incomplete Information*

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Abstract

We provide a necessary and sufficient condition for the existence of a stationary perfect Bayesian equilibrium (SPBE) of a non-cooperative bargaining game model in which all types of proposers offer the ex post efficient, Bayesian incentive compatible, budget-balanced mechanism with the full "residual" surplus extraction property. We define the asymmetric Nash bargaining solution (ANBS) under incomplete information which is realized as the limit of SPBEs when the risk of breakdown is vanishing. We also show that such a convergence results does not necessarily hold because of incomplete information.

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Key words: Nash bargaining solution, incomplete information, informed principal, non-cooperative bargaining

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1 Introduction

This paper examines the relationships between the equilibrium outcome of a noncooperative bargaining game and the Nash bargaining solution (NBS) under incomplete information. We extend a n-person complete information bargaining game model in which the correspondence between the stationary subgame perfect equilibrium (SSPE) and the asymmetric NBS exists to the bargaining problem with incomplete information. We consider a variation of a noncooperative bargaining game model with random proposers by Hart and Mas-Colell (1996) and Okada (1996). The bargaining procedure is described as follows: One player is selected as a proposer according to some probability distribution among n players. The selected player proposes a feasible allocation rule, i.e., a "mechanism". If the proposal is accepted unanimously, all players play a communication game under the mechanism. If some player rejects the proposal, the game ends with some exogenously given probability of breakdown. With the complementary probability, the game goes to the next round.

We consider a bargaining problem with incomplete information, in which each player has a private information about his type and proposes a mechanism when he becomes a proposer. Thus, an informed player designs a mechanism about bargaining outcomes. Therefore, our bargaining game includes the problem of mechanism design by an informed principal in Myerson (1983).

Since the seminal work by Nash (1953), there is a vast number of studies on the relationships between an equilibrium outcome of a noncooperative bargaining game and an NBS under complete information. Rubinstein (1982) provides the alternating-offers bargaining game where the payoff allocations in every subgame perfect equilibrium (SPE) converge to the NBS in the limit as players become perfectly patient. Binmore, Rubinstein and Wolinsky (1986) obtain the Nash bargaining solution in the limit if the exogenous risk of breakdown is vanishing. Binmore (1987) obtains the asymmetric NBS as an SPE outcome in the bargaining game with a generalized probability distribution under which a player is selected as a proposer.

The extensions to the n-person NBS has been pursued by Hart and Mas-Colell (1996) and Krishna and Serrano (1996). Krishna and Serrano (1996) provides a noncooperative bargaining game model in which players can exit after partial agreements and their bargaining procedure does not contain the chance moves and stochastic elements. On the other hand, in Hart and Mas-Colell (1996), a proposer is randomly selected with equal probability and the proposal is agreed to by unanimous consent among the players. If the proposal is rejected by some players, players face a risk of breakdown of the negotiations to continue the next bargaining round. Our bargaining game model is an extension of Hart and Mas-Colell's model to a general probability distribution. Recently, some noncooperative multilateral bargaining game models are provided to support the n-person asymmetric NBS by Miyakawa (2008), Okada (2007), Laruelle and Valenciano (2008), Kultti and Vartiainen (2010), and Britz, Herings and Predtetchinski (2010). Our bargaining game model is reduced to the model in Miyakawa (2008) if the game is in complete information.

The NBS for the bargaining problem with incomplete information has been examined by Harsanyi and Selten (1972), Myerson (1984), Weidner (1992) and de Clippel and Minelli (2004). Harsanyi and Selten derived the generalized NBS in incomplete information bargaining problem as a Bayesian Nash equilibrium of a noncooperative game satisfying axioms. Myerson defined the generalization of the NBS using three axioms: a probability-invariance axiom, a variant axiom of independence of irrelevant alternatives, and a random-dictatorship axiom. Weidner considered the relationships between two studies. de Clippel and Minelli considered the incomplete information bargaining game with verifiable types. All these studies refine the set of Bayesian Nash equilibria of the noncooperative game using some axioms. We focus on a noncooperative foundation of the NBS rather than the refinement through axioms. Thus, we adopt an appropriate extensive form game of bargaining to have a relationship to the NBS under incomplete information. In addition, we consider n-person, precisely more than 2 person, bargaining game problem. The previous studies examined only a 2 person bargaining game. Recently, Okada (2009) investigates the concept of core under incomplete information and the n-person noncooperative bargaining game with verifiable types and with coalition formations. We exclude the problem of coalition formation, but consider the game with unverifiable types.

It is noteworthy to mention the reason why we consider more than 2 player game. Our purpose is not a generalization of the number of players in a bargaining game. In general, the NBS under incomplete information necessitates the interim efficient (IE) mechanism to realize the NBS allocation. However, as Myerson and Satterthwaite (1983) pointed out, even a ex post efficient (EPE) mechanism fails to exist, satisfying Bayesian incentive compatibility (BIC), interim individual rationality (IR) and ex post budget balance (BB). To achieve the NBS under incomplete information as an equilibrium outcome, we need to consider a situation where at least an EPE mechanism always exists in the bargaining game. Precisely, we assume that the joint probability distribution of types satisfies Cremer-McLean condition in Cremer and MacLean (1988) and McAfee and Reny (1992) and Identifiability condition in Kosenok and Severinov (2008) and Severinov (2008). Under these assumptions, types of players must be correlated and there must be more than 2 players in the bargaining game to satisfy both conditions.

We obtain the following results. We provide a necessary and sufficient condition for the existence of an SPBE of the noncooperative bargaining game in which every player proposes the EPE, BIC, BB mechanism satisfying the "full surplus extraction" property, as examined in Cremer and McLean (1988), McAfee and Reny (1992), Kosenok and Severinov (2008) and Severionv (2008). Moreover, the proposals in the SBPE is *nuetral optimum* (Myerson, 1983) for the proposer. As a risk of the breakdown of negotiations is vanishing, the limit of the conditionally expected payoff vector in the SPBE is characterized as the asymmetric NBS under incomplete information. The asymmetric NBS is different from the NBSs in Harsanyi and Selten (1972) and Myerson (1984). In the previous studies under complete information, the equilibrium outcomes and all players' proposals converge to the NBS uniquely as a risk of breakdown is vanishing or as all players are patient enough. Recently, Kultti and Vartiainen (2010) and Herings and Predtetchinski (2010) have shown that the convergence result does not hold if the boundary of the set of feasible payoffs is not differential. We show that the convergence result fails to hold under incomplete information too.

This paper is organized as follows. Section 2 defines the Bayesian bargaining problem and the Nash bargaining solutions under incomplete information. Section 3 provides a noncooperative bargaining game with incomplete information. Section 4 examines the relationships to the literature in mechanism design by informed principal. Section 5 characterizes an SPBE of the noncooperative bargaining game. Section 6 discusses relationships between the SPBE and the NBS under incomplete information. Section 7 concludes.

2 Nash Bargaining Solution

2.1 Bayesian Bargaining Problem

We consider n-person bargaining problem with n (≥ 3) private informed players. We denote the set of players by $N = \{1, 2, \ldots, n\}$ and a generic element by $i \in N$. As in Myerson (1983, 1984), a n-person bargaining problem Γ is characterized by the following form

$$\Gamma = (D, d^*, \{\Theta_i\}_{i \in N}, \{v_i\}_{i \in N}, p),$$

where D is the finite set of public decisions or feasible outcomes and $d^* \in D$ is the disagreement point. For each player i, Θ_i is the set of possible types and θ is a generic element of Θ_i. We also denote the set of type profile by $\Theta = \prod_{j \in N} \Theta_j$ and a element by $\theta \in \Theta$. We let Θ_{-i} denote the set of types of the players other than i and $\theta_{-i} \in \Theta_{-i} = \prod_{j \neq i} \Theta_j$. We assume that D and Θ are finite sets[1].

Probability measure p is a common prior on Θ and $p_i(\theta_i)$ denote the marginal probability distribution of player i's type θ_i. The probability distribution of type profile θ_{-i} of players other than player i conditional on type θ_i of player i is

$$p_i(\theta_{-i}|\theta_i) = \frac{p(\theta)}{\sum_{\theta'_{-i} \in \Theta_{-i}} p(\theta'_{-i}, \theta_i)}.$$

Each v_i is a payoff function from $D \times \mathbb{R} \times \Theta$ to the real number $\mathbb{R}$. We assume that a payoff function for each player i is quasi-linear in decision d

[1]It is well-known that no a priori finite bound on the number of types exists to model a game with incomplete information (Mertens and Zamir, 1985). Moreover, it should be assumed that the type space has the "beliefs-determine-preferences" property, thus, there is a one-to-one correspondence between a player's preferences and a player's beliefs about other types. Heifetz and Neeman (2006) pointed out that information structures with this property are "small" among all conceivable common prior information structure.

and transfer t_i, i.e., $v_i(d, t_i, \theta) = u_i(d, \theta) + t_i$. A payoff for each player in disagreements is normalized to zero. That is, it is assumed that $v_i(d^*, 0, \theta) = u_i(d^*, \theta) = 0$ for all $\theta \in \Theta$.

2.2 Feasible and efficient mechanism

A mechanism μ is formally defined as a combination of message spaces $S_1, \ldots, S_n$ for all players and an outcome function $g : \prod_{i \in N} S_i \to D \times \mathbb{R}^n$ which maps from the set of message profiles to the set of public decisions and transfers. We write $\mu = (S_1, \ldots, S_n, g) \in \mathfrak{M}$ and $g(\cdot) = (d(\cdot), t(\cdot))$, where $\mathfrak{M}$ is the set of mechanisms. We call a mechanism in which the message space S_i for each player is her type space Θ_i itself *direct mechanism.* A direct mechanism is represented by $\mu = (\Theta_1 \times \cdots \times \Theta_n, g(\cdot))$, where $g : \Theta_1 \times \cdots \times \Theta_n \to D \times \mathbb{R}^n$. Let us denote $g(\cdot) = (d(\cdot), t(\cdot))$. We call $d : \Theta \to D$ the decision rule and $t : \Theta \to \mathbb{R}^n$ the transfer rule. We assume that all mechanisms in the set of mechanisms $\mathfrak{M}$ have a finite set of outcomes. Moreover, without loss of generality, we focus on deterministic outcome functions because the payoff function is quasi-linear.

For any direct mechanism μ, we can define the conditionally expected payoff for player i, given that his type is θ_i, if all players report their types truthfully as follows:

$$U_i(\mu|\theta_i) = \sum_{\theta_{-i} \in \Theta_{-i}} [u_i(d(\theta_{-i}, \theta_i), (\theta_{-i}, \theta_i)) + t_i(\theta_{-i}, \theta_i)]\, p_i(\theta_{-i}|\theta_i).$$

Moreover, the conditionally expected payoff when he reports $\hat{\theta}_i \in \Theta_i$ and all other players report their types honestly is represented by

$$U_i(\mu, \hat{\theta}_i|\theta_i) = \sum_{\theta_{-i} \in \Theta_{-i}} \left[u_i(d(\theta_{-i}, \hat{\theta}_i), (\theta_{-i}, \theta_i)) + t_i(\theta_{-i}, \hat{\theta}_i)\right] p_i(\theta_{-i}|\theta_i).$$

Let us introduce some notions about the direct mechanism.

Definition 1. A direct mechanism μ is *Bayesian incentive compatible* (BIC) if for all $i \in N$ and for all $\hat{\theta} \in \Theta_i$,

$$U_i(\mu|\theta_i) \geq U_i(\mu, \hat{\theta}_i|\theta_i).$$

Definition 2. A direct mechanism $\mu(\cdot) = (d(\cdot), t(\cdot))$ is *budget balanced* (BB) if for all $\theta \in \Theta$,

$$\sum_{i \in N} t_i(\theta) = 0.$$

Definition 3. A decision rule $d(\cdot)$ is *ex post efficient* if for all $\theta \in \Theta$,

$$d(\theta) \in \arg\max_{d \in D} \sum_{i \in N} u_i(d, \theta).$$

Moreover, a mechanism $\mu(\cdot) = (d(\cdot), t(\cdot))$ is *ex post efficient* (EPE) if $\mu(\cdot)$ is BB and $d(\cdot)$ is EPE for all $\theta \in \Theta$.

Definition 4. A mechanism μ is *interim efficient* (IE) in $\mathfrak{M}$ if it is a solution to the maximization problem:

$$\max_{\mu \in \mathfrak{M}} \sum_{i \in N} \sum_{\theta_i \in \Theta_i} \zeta_i(\theta_i) U_i(\mu|\theta_i),$$

where $\zeta_i(\theta_i)$ is a positive number for each player i and each type θ_i.

The utility weights for each player depend only on the player's own type in the maximization problem for the IE mechanisms. As Holmstrom and Myerson (1983) have argued, the set of IE mechanisms in $\mathfrak{M}$ is a subset of the set of EPE mechanisms in $\mathfrak{M}$.

2.3 Nash bargaining solution under incomplete information

Let us introduce the concept of the NBS under incomplete information. Focus on the set of all incentive compatible mechanisms. The conditionally expected payoff for each player is defined according to truthful reports.

Given any mechanism μ, we let $U(\mu)$ denote the vector of all conditionally expected payoffs $U_i(\mu|\theta_i)$ for each type of each player. That is, $U(\mu) = ((U_i(\mu|\theta_i))_{\theta_i \in \Theta_i})_{i \in N}$.

Harsanyi and Selten (1972) proposed the generalized NBS for games with incomplete information as a solution of

$$\max_{\mu \in \mathfrak{M}} \prod_{i \in N} \left(\prod_{\theta_i \in \Theta_i} U_i(\mu|\theta_i)^{p_i(\theta_i)} \right),$$

where $p_i(\theta_i) = \sum_{\theta_{-i} \in \Theta_{-i}} p(\theta_{-i}, \theta_i)$. We call it Harsanyi-Selten solution. Nash (1950) presented the symmetric NBS to a bargaining problem under complete information as a solution of

$$\max_{v \in V} \prod_{i \in N} (v_i),$$

where V is the set of feasible payoff allocations. Harsanyi and Selten solution is one of natural generalizations of the Nash (1950) bargaining solution.

The symetric NBS under complete information can be extended to the asymmetric NBS.

Definition 5. A payoff allocation v^* is called the *asymmetric Nash bargaining solution (ANBS) under complete information* with positive weight $w = (w_1, \ldots, w_n) \in \Delta(N)$ if v^* is a solution of the maximization problem:

$$\max_{v \in V} \prod_{i \in N} (v_i)^{w_i}, \tag{1}$$

where V is the set of feasible payoff allocations.

We introduce the ANBS with weight w under incomplete information, which is different from that in Harsanyi and Selten (1972).

Definition 6. The vector of all conditionally expected payoffs $U(\mu)$ is the *ANBS under incomplete information* with weight $w \in \Delta(N)$ to a Bayesian bargaining problem Γ if $U(\mu)$ is a solution of the maximization problem:

$$\max_{\mu \in \mathfrak{M}} \prod_{i \in N} \left(\sum_{\theta_i \in \Theta_i} p_i(\theta_i) U_i(\mu|\theta_i) \right)^{w_i},$$

where $\mathfrak{M}$ is the set of all feasible mechanisms.

We will show that the above ANBS under incomplete information is supported by a standard noncooperative bargaining game.

We have the following fundamental result:

Theorem 1. *Any mechanism to implement the ANBS under incomplete information with weight w is interim efficient.*

Proof. The maximization problem in the ANBS is transformed into the maximization problem of the form

$$\max_{\mu \in \mathfrak{M}} \prod_{i \in N} \sum_{\theta_i \in \Theta_i} \zeta_i(\theta_i) U_i(\mu|\theta_i),$$

where

$$\zeta_i(\theta_i) = \frac{w_i p_i(\theta_i)}{\sum_{\theta_i' \in \Theta_i} p_i(\theta_i') U_i(\mu|\theta_i')}.$$

Thus, the mechanism corresponding to the ANBS under incomplete information is IE by Definition 4. □

Remark 1. Let us consider a example of public project which was discussed in Myerson (1979, 1984) in order to understand the relationships between Harsanyi-Selten solution and the ANBS under incomplete information with weight w. For simplicity, we assume that $w_i = \bar{w}$ for all $i \in N$. Two players face a decision whether to build a public project which costs \$100. Player 1 has two possible types of the valuation for the public project. If player 1 is $1h$ type, which is denoted by θ_1^h, then the public project is worth \$90 to him and the event of $\theta_1^h = 90$ occurs with probability 9/10. If player 1 is 1ℓ type; θ_1^ℓ, then the public project is only worth \$30 to him and the event of $\theta_1^\ell = 30$ occurs with probability 1/10. Player 2 has only one possible type and the public project is always worth \$90 to her. Thus, $\theta_2 = 90$. Formally, $\Theta_1 = \{\theta_1^h, \theta_2^\ell\} = \{90, 30\}$, $\Theta_2 = \{90\}$, $d = 0, 1$ and $D = \{(d,t) \in \{0,1\} \times \mathbb{R}^2 | t_1 + t_2 \geq 100d\}$. A payoff function for player i is given by $u_i((d,t);\theta_i) = \theta_i d - t_i$. The set of interim utilities which can be achieved by feasible BIC mechanism is represented by the set of vectors

$$\begin{aligned}&(U_1(\mu|\theta_1^h), U_1(\mu|\theta_1^\ell); U_2(\mu|\theta_2))\\&= (90d(30) - t_1(30), 30d(30) - t_1(30); 72 - 82d(30) + t_1(30))\end{aligned}$$

with $d(30) \in [0,1]$ and $t_1(30) \in \mathbb{R}$.

In the ANBS under incomplete information, $\prod_{i=1}^2 \sum_{\theta_i \in \Theta_i} p_i(\theta_i) U_i(\mu|\theta_i)$ is maximized. Therefore, the solution to the maximization problem is given by the following pooling mechanisms:

$$d(30) = d(90) = 1, \quad t_1(30) = t_1(90), \quad t_2(30) = t_2(90) = 100 - t_1(30).$$

The expected utilities are $90 - t_1(30)$ for player 1 with type θ_1^h, $30 - t_1(30)$ for player 1 with type θ_1^ℓ and $90 - 100 + t_1(30)$ under $t_1(30) \in [10, 30]$ for player 2. Thus, the ANBS under incomplete information consists of multiple payoff allocations in this example. On the other hand, the Harsanyi-Selten solution is given by the vector of interim utilities

$$(U_1(\mu^{\text{HS}}|\theta_1^h), U_1(\mu^{\text{HS}}|\theta_1^\ell); U_2(\mu^{\text{HS}}|\theta_2)) = (27, 486/13; 36).$$

A *neutral bargaining solution* (Myerson, 1984), which is a bargaining solution satisfying the *probability-invariance* axiom, the *extension* axiom and the *random-dictatorship* axiom, is given by the vector of utilities

$$(U_1(\mu^N|\theta_1^h), U_1(\mu^N|\theta_1^\ell); U_2(\mu^N|\theta_2)) = (40, 10; 36).$$

These three solutions have a distinct value in this example.

3 Non-cooperative Bargaining Game

We present a noncooperative bargaining game model to realize the ANBS under incomplete information as an equilibrium outcome. The key feature of our bargaining game is that a player who is selected as a proposer offers a mechanism to determine a public decision and transfers among players and, then, all other players accept or reject the mechanism. Thus, players negotiate about a mechanism in the bargaining game.

We consider the following noncooperative bargaining game $G(\Gamma, w, \rho)$.

Stage 0: A nature selects a type profile $\theta \in \Theta$. Each players learn his own types θ_i privately.

Stage 1: At the beginning of each round t, one player is selected as a proposer according to a probability distribution $w \in \Delta(N)$. w_i is a probability that player i is chosen as a proposer among N.

Stage 2: The selected proposer i offers a mechanism $\mu^i \in \mathfrak{M}$.

Stage 3: All other players accept or reject the mechanism simultaneously.

Stage 4: If all players accept it, μ^i is implemented, i.e., each player sends a message $s_i \in S_i$ and then, the outcome $g(s) \in D \times \mathbb{R}^n$ is determined. If some player rejects it, the game continues to stage 1 in the next round with probability ρ. Otherwise, the negotiation breaks down with probability $1 - \rho$ and the game ends. When the game ends, all players get their disagreement payoff of 0.

The bargaining game is regarded as an extension of the informed principal game by Myerson (1983). If a proposer is predetermined and the game always ends when the proposal is rejected, i.e., $\rho = 0$, our bargaining game is the same as in Myerson (1983). If the game is in complete information, our game is reduced to a bargaining game model in Miyakawa (2008) and Okada (2007).

We adopt a perfect Bayesian equilibrium (PBE) (Fudenberg and Tirole, 1991, 1993) with a stationary property as a solution concept. When the game is with complete information, the solution concept corresponds to a stationary subgame perfect equilibrium (SSPE).

The bargaining game model can be represented by an infinite-length extensive form game. All nodes in an information set of player i in the extensive form at round t is determined by a sequence of past actions $z = (z_1, \dots, z_{t-1}, z_t)$, where z_t, $t = 1, 2, \dots$, denotes the sequence of actions in round t. It describes a history about who became a proposer, what a mechanism was offered by the proposer and which of an acceptance or a rejection each responder selected. A posterior belief $\beta_i(\theta_i)$ about other players' types for player i in type θ_i is represented by a probability measure on Θ_{-i}. The beliefs for all players is denoted by $\{\beta_i\}_{i \in N} = \{\{(\beta_i(\theta_i))_{\theta_i \in \Theta_i}\}_{i \in N}\}$, where $\beta_i(\theta_i) \in \Delta(\Theta_{-i})$. As a result, a state at round t is given by $(z, \{\beta_i\}_{i \in N})$. We denote a strategy for player i by a sequence $\sigma_i = \{\sigma_i^t\}_{t=0}^{\infty}$, where σ_i^t is the tth round strategy. A strategy combination $\sigma = (\sigma_1, \dots, \sigma_n)$ determines the payoffs for all players.

Definition 7. A pair of a strategy combination and a belief system (σ, β) is called a *stationary perfect Bayesian equilibrium* (SPBE) if σ is an PBE and σ_i^t in each bargaining round t ($t = 1, 2, \dots$) depends only on a belief system β_i and history z_t within round t.

In an SPBE, every player's action does not depend on the whole history of actions. Moreover, any player's behavior in each bargaining round does not change even if agreements were rejected in past rounds.

4 Informed Principal Mechanism Design

4.1 Cremer-McLean, Kosenok-Severinov condition

Our goal is to show that our noncooperative bargaining game model realizes the ANBS under incomplete information as an SPBE outcome. As in Theorem 1, a mechanism to implement the NBS under incomplete information must be IE. Therefore, we focus on the situation where there exists at least an EPE mechanism to reach our goal. However, it is well known that mechanisms satisfying BIC, interim IR, ex post BB and EPE can fail to exist in private values environments with independent types, as Myerson and Sattherthwaite (1983) have shown. We impose the following conditions on the prior distribution p on the type space Θ in order to ensure the existence of EPE mechanisms. The first one is introduced by Cremer and McLean (1988), so it is called "Cremer-McLean condition."

Definition 8. A probability distribution p satisfies *Cremer-McLean(CM) condition* if there are no $i \in N$, $\theta_i \in \Theta_i$ and $\lambda_i : \Theta_i \setminus \{\theta_i\} \to \mathbb{R}_+$ such that

$$p_i(\theta_{-i}|\theta_i) = \sum_{\theta_i' \in \Theta_i \setminus \{\theta_i\}} \lambda_i(\theta_i') p_i(\theta_{-i}|\theta_i'), \qquad \text{for all } \theta_{-i} \in \Theta_{-i}.$$

This condition means that vectors $p_i(\cdot|\theta_i)$ can not be expressed as a convex combination of all other vectors $p_i(\cdot|\theta_i')$, $\theta_i' \neq \theta_i$ with weights $\lambda_i(\theta_i')$.

We add identifiability condition by Kosenok and Severinov (2008).

Definition 9. A probability distribution p satisfies *identifiability(I) condition* if for all $q \in \Delta(\Theta)$; $q \neq p$, there exists $i \in N$ and $\theta_i \in \Theta_i$ such that $q_i(\theta_i) > 0$ and for any collection of nonnegative coefficients $\{\lambda_{\theta_i', \theta_i}\}$, we have

$$q_i(\theta_{-i}|\theta_i) \neq \sum_{\theta_i' \in \Theta_i} \lambda_{\theta_i', \theta_i} p_i(\theta_{-i}|\theta_i')$$

for at least one $\theta_{-i} \in \Theta_{-i}$.

Note that CM condition rules out the cases that types of players are independent and that each player's conditional beliefs are independent of his type. Thus, a prior p has some correlation among types. CM condition holds generically when the number of types for each player is less than or equal to the number of types of all other players. Moreover, as shown in Kosenok and Severinov (2008), I condition holds generically when there are at least three players ($n \geq 3$), where in case that $n = 3$, at least one of the players has at least three types. CM condition and I condition will ensure the existence of the EPE, interim IR, ex-post BB, BIC mechanism for each proposer in a bargaining game. This result has been established by Kosenok and Severinov (2008).

Example 1. Kosenok and Severinov (2008) have presented a sufficient condition for CM and I condition in the case with three players, each of whom has two types, to be satisfied. Let us denote $\Theta_i = \{\theta_i^1, \theta_i^2\}$ for $i \in \{1, 2, 3\}$. The condition is given by

$$p(\theta_1^1\theta_2^1\theta_3^1)p(\theta_1^1\theta_2^2\theta_3^2) - p(\theta_1^1\theta_2^1\theta_3^2)p(\theta_1^1\theta_2^2\theta_3^1) < 0,$$
$$p(\theta_1^1\theta_2^1\theta_3^1)p(\theta_1^2\theta_2^1\theta_3^2) - p(\theta_1^1\theta_2^1\theta_3^2)p(\theta_1^2\theta_2^1\theta_3^1) > 0.$$

The following joint probability of types in a game with three players, each of whom has two types satisfies the above conditions:

	θ_2^1	θ_2^2
θ_1^1	0.2	0.2
θ_1^2	0.1	0

θ_3^1

	θ_2^1	θ_2^2
θ_1^1	0.2	0.1
θ_1^2	0.2	0

θ_3^2

Figure 1: Example of joint probability distribution of types

A number in the cell of the matrix represents a probability of each combination of types. In the above example, $p(\theta_1^1, \theta_2^1, \theta_3^1) = 0.2$ and $p(\theta_1^2, \theta_2^1, \theta_3^1) = 0.1$.

4.2 Strong solution, neutral optimum and RSW allocation

Our bargaining game includes a mechanism design problem by informed principal. After a proposer in the bargaining game is selected, a proposer with his private information offers a mechanism and other players accept or reject the proposal. Now, assume that a proposer is player i and a feasible mechanism μ^0 is arbitrated if some player rejects the proposal. We focus on this part of the bargaining game, which is called the *informed principal game*. Let us clarify the relationships between the solution concept in the informed principal problem by Myerson (1983, 1984) and Maskin and Tirole (1992) and the equilibrium proposal in our bargaining game.

Myerson (1983) has provided a *strong solution* for the principal as a reasonable solution to the mechanism design problem. A mechanism is said to be *safe* for the principal if the mechanism is incentive feasible and would remain BIC and interim IR even if all the players knew the principal's true type, no matter what that type may be. A mechanism μ^i is said to be *undominated* if there is no mechanism ν^i such that all types of the principal i would expected at least a higher payoff in ν^i than in μ^i, thus, $U_i(\mu^i|\theta_i) \leq U_i(\nu^i|\theta_i)$ for all

$\theta_i \in \Theta_i$, with strict inequality for at least one $\theta_i \in \Theta_i$. A mechanism is a *strong solution* for the principal relative to μ^0 if it is safe for the principal and is undominated, and an outcome in disagreements is given by μ^0. However, a strong solution fails to exist for many informed principal games. Then, Myerson (1983) defined a *neutral optimum* as an alternative solution concept. Let $\Gamma_i = (D, \mu^0, \{\Theta_i\}_{i \in N}, \{v_i\}_{i \in N}, p)$ be the informed principal game with the principal i. Let $B(\Gamma_i)$ denote the set of blocked payoff allocations of the principal i, which satisfies the following axioms:

Axiom 1 (Domination)**.** For any $w, z \in \mathbb{R}^{\Theta_i}$, if $w \in B(\Gamma_i)$ and $\forall \theta_i \in \Theta_i$, $z(\theta_i) \leq w(\theta_i)$, then $z \in B(\Gamma_i)$.

Axiom 2 (Openness)**.** For all Γ_i, $B(\Gamma_i)$ is open subset of $\mathbb{R}^{\Theta_i}$.

We say that $\bar{\Gamma}_i = (\bar{D}, \mu^0, \{\Theta_i\}_{i \in N}, \{\bar{v}_i\}_{i \in N})$ is a extension of Γ_i if $D \subseteq \bar{D}$ and $\bar{v}_i(d, \theta) = v_i(d, \theta)$ whenever $d \in D$.

Axiom 3 (Extension)**.** If $\bar{\Gamma}_i$ is any extension of Γ_i, then $B(\Gamma_i) \subseteq B(\bar{\Gamma}_i)$.

Axiom 4 (Strong Solution)**.** If μ^i is a strong solution for the principal i relative to μ^0, then $(U_i(\mu^i|\theta_i))_{\theta_i \in \Theta} \notin B(\Gamma_i)$.

Let us define $B^*(\Gamma_i) = \bigcup_{B(\Gamma_i) \in H} B(\Gamma_i)$, where H is the set of all functions satisfying Axiom 1-4.

Definition 10. A mechanism μ^i is a *neutral optimum* for the principal i relative to μ^0 in the informed principal game Γ_i if it is both BIC and IR and $(U_i(\mu^i|\theta_i))_{\theta_i \in \Theta_i} \notin B^*(\Gamma_i)$.

See, also, Severinov (2008). As Myerson (1983) has shown, a neutral optimum always exists for any informed principal game, and if a strong solution exists, then it is a neutral optimum. We will see later that the proposal by player i in the candidate strategy profile of the bargaining game is a neutral optimum for the principal i.

Maskin and Tirole (1992) defined an RSW (Rothschild-Stiglitz-Wilson) allocation $\hat{\mu}^i(\mu^0)$ relative to the reservation allocation μ^0 as a mechanism that maximizes the payoff of each type of principal among the class of BIC mechanisms that guarantee the agents at least the utilities in μ^0 and still BIC even if the agents knew the principal's type[2]. As Maskin and Tirole mentioned, an RSW allocation is a strong solution for the principal if and only if it is interim efficient relative to prior beliefs p. Therefore, when a strong solution exists, a neutral optimum mechanism coincides with an RSW allocation.

5 Characterization of Equilibria

5.1 Inscrutability Principle

Let us start to characterize a SPBE in our bargaining game model.

First, there is no loss of generality in considering only direct incentive compatible mechanisms on the equilibrium path of $G(\Gamma, w, \rho)$ by the *revelation principle* in Myerson (1979). For any mechanism $\mu^i \in \mathfrak{M}$ which is proposed by player i in any PBE, there exists an outcome-equivalent direct BIC mechanism.

Second, there is no loss of generality in assuming that all types of the proposer should offer the same mechanism on the equilibrium path because there exists a type-independent mechanism which implement a same outcome by a combination of mechanisms such that different types of the principal

[2]In Myerson and Tirole (1992), an RSW allocation is defined for 2-person game. We consider an RSW allocation which is naturally extended to n-person game.

prefer different mechanisms. As a result, the proposer's actual choice of mechanism seems to convey no information about the type of the proposer to other players on the equilibrium path. This assumption is called the *inscrutability principle* by Myerson (1983).

Owing to above principles, we can prove the existence of SPBE of the bargaining game in a simple manner. We can assume that on the SPBE path of $G(\Gamma, w, \rho)$, all types of the proposer i offer a same direct mechanism $(x^i(\cdot), t^i(\cdot))$ which is incentive compatible under the beliefs $p_j(\theta_{-j}|\theta_j)$ in the initial round. Moreover, the equilibrium beliefs of any player at stage 2 in the initial round are equal to $p_j(\theta_{-j}|\theta_j)$ by the inscrutability principle. Even in sequential bargaining rounds, all types of the proposer offer the same direct incentive compatible mechanism under the beliefs at the beginning of the bargaining round and the responders' beliefs are unchanged on the equilibrium path. We will show that there exists an SPBE such that every proposal is accepted in the initial round and the belief system $\{\beta_i\}_{i \in N}$ on the equilibrium path remains unchanged at the initial posterior belief system $\{(p_i(\theta_{-i}|\theta_i))_{\theta_i \in \Theta_i}\}_{i \in N}$ by the Bayes's rule.

5.2 Existence of perfect Bayesian Equilibrium

In this section, we show that our noncooperative bargaining game $G(\Gamma, w, \rho)$ has a SPBE in which each proposer offers a mechanism with the full "residual" surplus extraction property. Here, the full "residual" surplus extraction property means that a proposer gets all residual surplus after giving only their expected continuation payoffs of all responders if they reject the proposal. The proposal with this property has been played an important role in the noncooperative bargaining game theory. For example, consider the ultimatum game. It is well known that the continuation payoff of a responder is zero because the game ends if he rejects the proposal. Therefore, the proposer offers a proposal to extract all surplus of their cooperation and this proposal consists of a subgame perfect equilibrium. Even in Rubinstein's alternating-offer bargaining or other bargaining game models, a player offers a proposal to assign the responders only their continuation payoffs when they reject the proposal. If the residual surplus by any acceptable proposal is negative, the proposer selects a delay of agreement.

We apply the same idea to the bargaining game with incomplete information. In the context of mechanism design, the full surplus extraction has been examined by Cremer and McLean (1988), McAfee and Reny (1992) and Kosenok and Severinov (2008). They identified a necessary and sufficient condition for the full surplus extraction by the uninformed principal through BIC, IR, EPE mechanisms with or without ex-post BB, which is *CM condition* and *I condition*. Severinov (2008) showed that there exists an EPE, interim IR, ex-post BB, BIC mechanism with full surplus extraction property if a prior distribution about types satisfies CM and I condition even in the informed principal setting. Here, IR implies the requirement for the acceptance of the proposal by each responder. We will examine an PBE of $G(\Gamma, w, \rho)$ such that every proposer offers the proposal with the full "residual" surplus extraction property.

In order to formalize a proposal with the full "residual" surplus extraction property, let us firstly define the expected social surplus from an EPE mechanism for type θ_i of player i by

$$W_i(\theta_i) = \sum_{\theta_{-i} \in \Theta_{-i}} \left[\max_{d \in D} \sum_{j \in N} u_j(d, (\theta_{-i}, \theta_i)) \right] p_i(\theta_{-i} \mid \theta_i).$$

We impose the following assumption about the expected social surplus:

Assumption 1. The expected social surplus from an EPE mechanism for every type of every player is strictly positive; i.e., $W_i(\theta_i) > 0$ for all $\theta_i \in \Theta_i$ and $i \in N$.

By Corollary 1 of Theorem 1 in Kosenok and Severinov (2008), we have the existence theorem of EPE, BIC, BB mechanisms as follows:

Theorem 2. (Kosenok and Severinov, 2008) *Under CM and I condition, there exists an EPE, BIC, BB mechanism $\mu^{i*} = (d^{i*}(\cdot), t^{i*}(\cdot))$ to realize a nonnegative payoff vector $((R_i^i(\theta_i))_{\theta_i \in \Theta_i}, ((V_j^i(\theta_j))_{\theta_j \in \Theta_j})_{j \in N, j \neq i})$ such that the expected payoff $R_i^i(\theta_i)$ of type θ_i of player i is equal to*

$$R_i^i(\theta_i) = W_i(\theta_i) - \sum_{\theta_{-i} \in \Theta_i} p(\theta_{-i}|\theta_i) \sum_{j \in N, j \neq i} \rho v_j(\theta_j)$$

and the expected payoff $V_j^i(\theta_j)$ for each type of player $j(\neq i)$ is $\rho v_j(\theta_j)$.

Proof. See Appendix. □

Because $\rho v_j(\theta_j)$ is considered as the continuation payoff for type θ_j of player j, the mechanism μ^{i*} corresponds to the proposal with the full "residual" surplus extraction property by player i.

We will show that the following strategies and beliefs can be supported as a part of PBE of $G(\Gamma, w, \rho)$: First, all types of player i offer the mechanism μ^{i*} at stage 2 in the initial bargaining round if he is selected as a proposer. Then, all types of all other players accept the proposal in stage 3. In stage 4, all players report their types truthfully. Thus, the mechanism μ^{i*} is implemented. Beliefs in stage 3 after the mechanism μ^{i*} is proposed are equal to the initial conditionally beliefs $p_i(\cdot|\theta_i)$ for any type θ_i of player $i \in N$. Moreover, beliefs in stage 4 when they report their types after all types accept the proposal μ^{i*} are also equal to the initial beliefs.

Theorem 3. *Suppose that probability distribution p satisfies CM and I conditions for all $i \in N$. There exists an SPBE of $G(\Gamma, w, \rho)$ in which all types* *of player i as a proposer offer mechanism μ^{i*} and the proposal is accepted at the initial round if and only if the equations system; for all $i \in N$ and for all $\theta_i \in \Theta_i$,*

$$v_i(\theta_i) = w_i \left[W_i(\theta_i) - \sum_{\theta_{-i} \in \Theta_{-i}} p_i(\theta_{-i}|\theta_i) \sum_{j \in N, j \neq i} \rho v_j(\theta_j) \right] + (1 - w_i)\rho v_i(\theta_i), \tag{2}$$

has nonnegative solution $((v_i^*(\theta_i))_{\theta_i \in \Theta})_{i \in N}$.

Proof. See Appendix. □

Applying the same argument in Severinov (2008) to a part of informed principal game in our model, we can show that the mechanism μ^{i*} for each proposer i is a neutral optimum. Let $\tilde{\mu}$ be the mechanism in which μ^{i*} is implemented with probability w_i for each $i \in N$.

Theorem 4. *Mechanism μ^{i*} is a neutral optimum relative to $\tilde{\mu}$ for the principal i.*

Proof. See Appendix. □

Theorem 4 says that the SPBE proposal for each player in Theorem 3 is stable from the notion of blocking in Myerson (1983). Moreover, if the strong solution exists, μ^{i*} is a strong solution relative to $\tilde{\mu}$ for the principal i.

6 Relationships to Nash Bargaining Solution

We clarify a relationship between the NBS and the expected payoff vector in the SPBE in Theorem 3.

6.1 Complete information case

First, let us consider a case in which the bargaining problem is under complete information. The bargaining game with complete information is regarded as a case that the type space is singleton; $\Theta = \{\theta\}$. A noncooperative bargaining game model corresponds to the model in Miyakawa (2008) and Okada (2007). In the singleton-type case, an SPBE implies an SSPE of the noncooperative bargaining game. Equation (2) is reduced to, for $i = 1, \ldots, n$,

$$v_i(\theta_i) = w_i \left[\max_{d \in D} \sum_{j \in N} u_j(d, \theta) - \sum_{j \in N \setminus \{i\}} \rho v_j(\theta_j) \right] + (1 - w_i)\rho v_i(\theta_i). \quad (3)$$

We denote the maximum aggregate payoff by $W(\theta) = \max_{d \in D} \sum_{j \in N} u_j(d, \theta)$. The equation system (3) has a nonnegative unique solution $(v_i^\rho(\theta_i))_{i \in N} = (w_i W(\theta))_{i \in N}$ for any ρ. Applying the same argument in Theorem 3, we have an SSPE of the bargaining game with complete information in which the expected payoff vector is $(v_i^\rho(\theta_i))_{i \in N}$. In the SSPE, player i proposes payoff allocation $(x_i^i(\theta_i), (\rho w_j W(\theta))_{j \in N \setminus \{i\}})$, where $x_i^i(\theta_i) = W(\theta) - \sum_{j \in N \setminus \{i\}} \rho w_j W(\theta)$. As the risk of breakdown is vanishing; $\rho \to 1$, the proposals by all players in the SSPE converge to the same payoff allocation. Assume that $\lim_{\rho \to 1} v_i^\rho(\theta_i) = v_i^*(\theta_i)$ for all $i \in N$. The limit SSPE payoff allocation satisfies

$$\frac{v_i^*(\theta_i)}{w_i} = \frac{v_j^*(\theta_j)}{w_j}, \quad \text{for all} \quad i, j \in N, i \neq j,$$
$$\sum_{i \in N} v_i^*(\theta_i) = \max_{d \in D} \sum_{i \in N} u_i(d, \theta).$$

This condition is identical to the Kuhn-Tucker condition of the maximization problem (1) for the ANBS under complete information.

Summarizing the above arguments, we have the following proposition:

Theorem 5. *Assume that $\Theta = \{\theta\}$. (i) There exists a unique SSPE of the noncooperative bargaining game with complete information $G(w, \rho)$ in which* *the SSPE payoff allocation is equal to the ANBS under complete information with weight w. (ii) As $\rho \to 1$, every proposal by the proposer converges to the ANBS under complete information with weight w and the proposal is accepted by all players immediately.*

Proposition 5 says that there is a one-to-one correspondence between the weight parameter w in the ANBS and the probability distribution w for the selection of a proposer in the noncooperative bargaining game. We call (ii) in Proposition 5 the *convergence result*.

6.2 Incomplete information case

Next, let us examine the bargaining game with incomplete information.

The case of $\rho = 0$: When $\rho = 0$, a proposer make a take-it-or-leave-it offer in the bargaining game, and the game ends with probability one if the proposal is rejected. Players play the same game as the informed principal game in Myerson (1983) after one player is selected as a proposer. (2) is reduced to

$$v_i(\theta_i) = w_i W_i(\theta_i), \quad i \in N, \theta_i \in \Theta_i.$$

Under Assumption 1, $v_i(\theta_i) \geq 0$. By Theorem 3, an SPBE of the bargaining game always exist. In the SPBE, the proposer i offers a mechanism which gives the proposer with type θ_i a payoff of $W_i(\theta_i)$ and the responders zero, which is denoted by μ^{i*M}, and the proposal is accepted. In this bargaining game, mechanism μ^{i*M} is implemented and player i extracts all surplus with probability w_i after the nature chooses a type $\theta \in \Theta$. The conditionally expected payoffs $((v_i^M(\theta_i))_{\theta_i \in \Theta_i})_{i \in N}$ in the SPBE satisfies

$$\frac{\sum_{\theta_i \in \Theta_i} p_i(\theta_i) v_i^M(\theta_i)}{w_i} = \frac{\sum_{\theta_j \in \Theta_j} p_j(\theta_j) v_j^M(\theta_j)}{w_j}, \text{ for } i, j \in N, i \neq j.$$

This means that the SPBE payoff allocation is fair in the ex ante sense because the all w-weighted ex ante expected payoffs are equal.

The case of $\rho \neq 0$: Although (2) has a nonnegative solution of $((v_j(\theta_j))_{\theta_j \in \Theta_j})_{i \in N}$ when ρ is close to zero, the value of $v_j(\theta_j)$ for some $\theta_j \in \Theta_j$ might be negative in the solution to (2) for large ρ. In this case, by Theorem 3, there is no SPBE in which player i proposes mechanism μ^{i*} and all responders accept the proposal immediately.

Before providing the main proposition, we re-examine the ANBS under incomplete information in Definition 6. By Corollary 1 in Kosenok and Severinov (2008), under CM and I conditions, there exists a BIC, BB, IR mechanism to realize the nonnegative expected payoff vector $((v_i(\theta_i))_{\theta_i \in \Theta_i})_{i \in N}$ which satisfies

$$\sum_{i \in N} \sum_{\theta_i \in \Theta_i} v_i(\theta_i) p_i(\theta_i) = \sum_{i \in N} \sum_{\theta \in \Theta} \max_{d \in D} u_i(d, \theta) p(\theta). \tag{4}$$

Thus, the set of feasible payoff allocation is the set of $((v_i(\theta_i))_{\theta \in \Theta})_{i \in N}$ satisfying (4). The ANBS under incomplete information is given by a solution to the maximization problem:

$$\max_{((v_i(\theta_i))_{\theta_i \in \Theta_i})_{i \in N}} \prod_{i \in N} \left(\sum_{\theta_i \in \Theta_i} p_i(\theta_i) v_i(\theta_i) \right)^{w_i} \tag{5}$$

$$\text{sub. to (4).}$$

A first order condition of the maximization problem is

$$\frac{\sum_{\theta_i \in \Theta} p_i(\theta_i) v_i(\theta_i)}{w_i} - \frac{1}{\lambda} = 0, \quad i = 1, \ldots, n, \tag{6}$$

and (4), where λ is a Lagrange multiplier. Thus, the maximization problem of the ANBS under incomplete information determines only the value of $\sum_{\theta_i \in \Theta_i} p_i(\theta_i) v_i(\theta_i)$ for each $i \in N$ and indetermines each $v_i(\theta_i)$.

As long as (2) has a nonnegative solution for any $\rho \in [0, 1)$, we have the following proposition about the convergence result:

Theorem 6. *Assume that (2) has a nonnegative solution of* $(v_i^\rho(\theta_i))_{\theta_i\in\Theta_i})_{i\in N}$ *for any* $\rho \in [0,1)$. *Let* $((v_i^*(\theta_i))_{\theta_i\in\Theta_i})_{i\in N}$ *be a limit point of* $(v_i^\rho(\theta_i)_{\theta_i\in\Theta_i})_{i\in N}$ *as* $\rho \to 1$. *Then, there exists a limit SPBE of the bargaining game* $G(\Gamma, w)$ *in which the expected payoff allocation is* $((v_i^*(\theta_i))_{\theta_i\in\Theta_i})_{i\in N}$ *and it belongs to the set of the ANBS under incomplete information. In the limit SPBE, all player propose the same mechanism to implement* $((v_i^*(\theta_i))_{\theta_i\in\Theta_i})_{i\in N}$ *and the proposal is accepted by all other players.*

Proof. Let us define

$$R_i^i(\theta_i) = W_i(\theta_i) - \sum_{\theta_{-i}\in\Theta_{-i}} p_i(\theta_{-i}|\theta_i) \sum_{j\in N, j\neq i} \rho v_j^\rho(\theta_j).$$

Rearranging (2), we have

$$R_i^i(\theta_i) = \frac{1-\rho}{w_i} v_i^\rho(\theta_i) + \rho v_i^\rho(\theta_i), \quad \text{for } \theta_i \in \Theta_i,\ i \in N. \tag{7}$$

By $\lim_{\rho\to 1} v_i^\rho(\theta_i) = v_i^*(\theta_i)$ and (7), we have that $\lim_{\rho\to 1} U_i(\mu^{i*}|\theta_i) = v_i^*(\theta_i)$ for all $\theta_i \in \Theta_i$ and for all $i \in N$ and also have that $\lim_{\rho\to 1} \rho v_i^\rho(\theta_i) = v_i^*(\theta_i)$. As $\rho \to 1$, all players proposes the same mechanism to realize $((v_i^*(\theta_i)_{\theta_i\in\Theta_i})_{i\in N}$ asymptotically in the SPBE. Thus, $\mu^{i*} \to \mu^*$ as $\rho \to 1$ for all $i \in N$ in the SPBE.

Because mechanism μ^{i*} is EPE, we have

$$R_i^i(\theta_i) + \sum_{\theta_{-i}\in\Theta_{-i}} p_i(\theta_{-i}|\theta_i) \sum_{j\in N, j\neq i} \rho v_j^\rho(\theta_j) = W_i(\theta_i), \quad \text{for all } \theta_i \in \Theta_i.$$

Multiplying the above equation by each $p(\theta_i)$ and adding them up together, we obtain

$$\sum_{\theta_i\in\Theta_i} p_i(\theta_i) R_i^i(\theta_i) + \sum_{j\in N, j\neq i} \sum_{\theta_j} p_j(\theta_j) \rho v_j^\rho(\theta_j) = \sum_{\theta\in\Theta} \left[\max_{d\in D} \sum_{j\in N} u_j(d,\theta)\right] p(\theta). \tag{8}$$

For other $j \in N$, $j \neq i$, we also have

$$\sum_{\theta_j \in \Theta_j} p_j(\theta_j) R_j^j(\theta_j) + \sum_{i \in N, i \neq j} \sum_{\theta_i} p_i(\theta_i) \rho v_i^\rho(\theta_i) = \sum_{\theta \in \Theta} \left[\max_{d \in D} \sum_{i \in N} u_i(d, \theta) \right] p(\theta). \tag{9}$$

Subtracting (9) from (8), we obtain

$$\sum_{\theta_i \in \Theta_i} p_i(\theta_i) \left(R_i^i(\theta_i) - \rho v_i^\rho(\theta_i) \right) - \sum_{\theta_j \in \Theta_j} p_j(\theta_j) \left(R_j^j(\theta_j) - \rho v_j^\rho(\theta_j) \right) = 0. \tag{10}$$

Substituting (7) to (10), we have

$$\frac{\sum_{\theta_i \in \Theta_i} p_i(\theta_i) v_i^\rho(\theta_i)}{w_i} = \frac{\sum_{\theta_j \in \Theta_j} p_j(\theta_j) v_j^\rho(\theta_j)}{w_j}, \quad \text{for } i, j \in N, i \neq j. \tag{11}$$

Because $\lim_{\rho \to 1} v_i^\rho(\theta_i) = v_i^*(\theta_i)$ and $R_i^i(\theta_i) = v_i^*(\theta_i)$, (8) and (10) is reduced to, as $\rho \to 1$,

$$\frac{\sum_{\theta_i \in \Theta_i} p_i(\theta_i) v_i^*(\theta_i)}{w_i} = \frac{\sum_{\theta_j \in \Theta_j} p_j(\theta_j) v_j^*(\theta_j)}{w_j}, \quad \text{for } i, j \in N, i \neq j, \tag{12}$$

$$\sum_{i \in N} \sum_{\theta_i \in \Theta} p_i(\theta_i) v_i^*(\theta_i) = \sum_{\theta \in \Theta} \max_{d \in D} \sum_{i \in N} u_i(d, \theta) p(\theta). \tag{13}$$

These conditions corresponds to the Kuhn-Tucker condition (6) and (4) of the maximization problem (5) for the ANBS under incomplete information. □

By (11), the SPBE payoff allocation is fair between players in the ex ante viewpoint, irrespective of ρ. Moreover, taking into account of (7), the SPBE selects an allocation from the set of the ANBS under incomplete information, which satisfies

$$\frac{v_i^*(\theta_i)}{v_i^*(\theta_i')} = \frac{R_i^i(\theta_i)}{R_i^i(\theta_i')} \quad \text{for } \theta_i, \theta_i' \in \Theta_i, \theta_i \neq \theta_i'.$$

Example 2. To understand the relationship between the ANBS under incomplete information and the SPBE of the bargaining game, let us consider the example of public project in Remark 1 again. Suppose that the game is played by three players and each player has two possible types; $N = \{1, 2, 3\}$ and $\Theta_i = \{\theta_i^1, \theta_i^2\}$. The joint probability distribution of types is same as in Example 1. For the simplicity, assume that $w_1 = w_2 = w_3 = 1/3$. The valuation of the public project is $\theta_1^1 = 30$ and $\theta_1^2 = 90$ for player 1, $\theta_2^1 = 40$ and $\theta_2^2 = 80$, and $\theta_3^1 = 70$ and $\theta_3^2 = 50$ for player 3. Under the joint probability distribution of types in Example 1, the marginal distributions of each type are $p_1(\theta_1^1) = 0.7$, $p_1(\theta_1^2) = 0.3$, $p_2(\theta_2^1) = 0.7$, $p_2(\theta_2^2) = 0.3$ and $p_3^1(\theta_3^1) = 0.5$, $p_3(\theta_3^2) = 0.5$. The set of payoff allocations in ANBS under incomplete information is given by

$$E = \left\{ (v_i(\theta_i^j))_{i=1,2,3}^{j=1,2} \;\middle|\; 0.7v_1(\theta_1^1) + 0.3v_1(\theta_1^2) = 70/3, \right.$$
$$\left. 0.7v_2(\theta_2^1) + 0.3v_2(\theta_2^2) = 70/3,\ 0.5v_3(\theta_3^1) + 0.5v_3(\theta_3^2) = 70/3 \right\}.$$

As $\rho \to 1$, the expected payoffs in the SPBE converge to $v_1^*(\theta_1^1) = 16/3$, $v_1^*(\theta_1^2) = 196/3$, $v_2^*(\theta_2^1) = 34/3$, $v_2^*(\theta_2^2) = 154/3$, $v_3(\theta_3^1) = 100/3$ and $v_3^*(\theta_3^2) = 40/3$. Therefore, the vector $(v_i^*(\theta_i^j))_{i=1,2,3}^{j=1,2}$ is in the set E. On the other hand, Harsanyi-Selten solution is given by $(v_i^{HS}(\theta_i^j))_{i=1,2,3}^{j=1,2} = (70/3, 70/3; 70/3, 70/3; 70/3, 70/3)$. The Harsanyi-Selten solution is not in E.

6.3 Failure of convergence result

As we have seen in Theorem 6, (2) must have a nonnegative solution for any $\rho \in [0, 1)$ to obtain the convergence result. However, under incomplete information, for some $\bar{\rho} \in [0, 1)$, for all $\rho \in [\bar{\rho}, 1)$, (2) might have a solution with some negative element $v_i^\rho(\theta_i)$. In this case, the convergence result does not hold because the PBE in Theorem 3 fails to exist for large ρ.

We provide a necessary and sufficient condition for the solution of (2) to be nonnegative. Let us denote $|\Theta_1| = K_1$, $|\Theta_2| = K_2$, $\ldots$, $|\Theta_n| = K_n$ and $K = \sum_{i \in N} K_i$, and let the type space of player i be $\Theta_i = \{\theta_i^1, \ldots, \theta_i^{K_i}\}$. Each equation in (2) is rewritten by, for $\theta_i \in \Theta_i$, $i \in N$,

$$\frac{1-(1-w_i)\rho}{w_i} v_i(\theta_i) + \sum_{j \in N\setminus\{i\}} \sum_{\theta_j \in \Theta_j} \frac{p(\theta_j, \theta_i)}{p_i(\theta_i)} \rho v_j(\theta_j) = W_i(\theta_i),$$

where $p(\theta_i, \theta_j) = \sum_{\theta_{-ij} \in \Theta_{-ij}} p(\theta_i, \theta_j, \theta_{-ij})$ is a marginal distribution of a pair of types (θ_i, θ_j). Therefore, the equation system (2) is rewritten in the matrix form:

$$Av = W, \tag{14}$$

where $A = [a_1, \ldots, a_K]$, $v = [v_1(\theta_1^1), \ldots, v_1(\theta_1^{K_1}), v_2(\theta_2^1), \ldots, v_n(\theta_n^{K_n})]^T$ and $W = [W_1(\theta_1^1), \ldots, W_n(\theta_n^{K_n})]^T$. Each a_i is a K-dimension vector as follows:

$$a_1 = \left[\frac{1-(1-w_1)\rho}{w_1}, 0, \ldots, 0, \frac{p(\theta_1^1 \theta_2^1)\rho}{p_1(\theta_2^1)}, \ldots, \frac{p(\theta_1^1, \theta_n^{K_n})\rho}{p_n(\theta_n^{K_n})}\right]^T$$

$$a_2 = \left[0, \frac{1-(1-w_1)\rho}{w_1}, 0, .., 0, \frac{p(\theta_1^1 \theta_2^1)\rho}{p_1(\theta_2^1)}, \ldots, \frac{p(\theta_1^1, \theta_n^{K_n})\rho}{p_n(\theta_n^{K_n})}\right]^T$$

$$\vdots$$

$$a_{K_1} = \left[0, ..., 0, \frac{1-(1-w_1)\rho}{w_1}, \frac{p(\theta_1^1 \theta_2^1)\rho}{p_1(\theta_1^1)}, \ldots, \frac{p(\theta_1^1, \theta_n^{K_n})\rho}{p_n(\theta_n^{K_n})}\right]^T$$

$$a_{K_1+1} = \left[\frac{p(\theta_2^1 \theta_1^1)\rho}{p(\theta_1^1)}, \ldots, \frac{p(\theta_2^1 \theta_1^{K_1})}{p_1(\theta_1^{K_1})}, \frac{1-(1-w_2)\rho}{w_2}, 0, \ldots, 0, \frac{p(\theta_2^1 \theta_n^1)\rho}{p_n(\theta_n^1)} \ldots, \frac{p(\theta_2^1 \theta_n^{K_n})\rho}{p_n(\theta_n^{K_n})}\right]^T$$

$$\vdots$$

$$a_K = \left[\frac{p(\theta_n^{K_n} \theta_1^1)\rho}{p_1(\theta_1^1)}, \ldots, \frac{p(\theta_n^{K_n} \theta_{n-1}^{K_{n-1}})\rho}{p_{n-1}(\theta_{n-1}^{K_{n-1}})}, 0, \ldots, 0, \frac{1-(1-w_n)\rho}{w_n}\right]^T.$$

Let $\langle x, y\rangle$ be the inner product of two vectors x and y. Applying Farkas' lemma (see, for example, Rockafellar, 1970) to (14) directly, we have

Lemma 1 (Farkas' lemma)**.** *There exists a nonnegative* $(v_1(\theta_1^1), \ldots, v_n(\theta_n^{K_n}))$ *such that* $Av = W$ *if and only if* $\langle W, x\rangle \leq 0$ *for all* x *such that* $\langle a_i, x\rangle \leq 0$ *for* $i = 1, \ldots, n$.

Let C denotes the set of all nonnegative linear combinations of $a_1, \ldots, a_K$. The condition in Lemma 1 is equivalent to $W \in \overline{C}$, where $\overline{C}$ is the closure of C.

If $\rho \to 0$, each vector a_i is close to the unit vector in which the ith element is 1, and then the set of $\overline{C}$ expands to the nonnegative orthant of K-dimensional space. Since $W_i(\theta_i) \geq 0$, there exists $\underline{\rho} \in [0, 1)$ such that for all $\rho \in [0, \underline{\rho})$ and for all $W \in \mathbb{R}^K$, $W \in \overline{C}$. This result is consistent with the fact that the candidate SPBE of the bargaining game always exist when the proposer makes a take-it-or-leave-it offer in the case of $\rho = 0$.

Next, consider a case in which all players is selected as a proposer with equal probability; $w_i = 1/n$ for all $i \in N$. In this case, the diversity among $W_i(\theta_i)$ for all θ_i is related to the convergence result. Let us consider the most extreme case such that $W_i(\theta_i) = \overline{W}$ for all $i \in N$ and all $\theta_i \in \Theta_i$. Thus, there is no diversity among $W_i(\theta_i)$. Since $1 - (1 - w_i)\rho/w_i = n - (n - 1)\rho > 1$ for all $i \in N$ and $0 \leq p(\theta_j, \theta_i)\rho/p_i(\theta_i) < 1$ for all $\theta_i \in \Theta_i$ and all $\theta_j \in \Theta_j$, the vector W is sure to be in the $\overline{C}$. The convergence result holds in this case.

Example 3: (Diversity of $W_i(\theta_i)$) We provide a example in which the convergence result fails to hold. Consider Example 2 again. However, assume that $\theta_1^1 = 90$, $\theta_1^2 = 30$, $\theta_2^1 = 80$, $\theta_2^2 = 40$ and others are same as Example 2. We have that $v_1^*(\theta_1^1) = 164/3$, $v_1^*(\theta_1^2) = -16/3$, $v_2^*(\theta_2^1) = 146/3$, $v_2^*(\theta_2^2) = 26/3$ and $v_3^*(\theta_3^1) = 146/3$, $v_3^*(\theta_3^2) = 80/3$. Thus, $v_1^*(\theta_1^2)$ is negative. In this example,

$$
\begin{aligned}
&(W_1(\theta_1^1), W_1(\theta_1^2), W_2(\theta_2^1), W_2(\theta_2^2), W_3(\theta_3^1), W_3(\theta_3^2)) \\
&= \left(\frac{87}{0.7}, \frac{23}{0.3}, \frac{79}{0.7}, \frac{31}{0.3}, \frac{61}{0.5}, \frac{49}{0.5}\right) \\
&\approx (124.2, 76.6, 112.8, 103.3, 122, 98).
\end{aligned}
$$

On the other hand, in Example 2,

$$
\begin{aligned}
&(W_1(\theta_1^1), W_1(\theta_1^2), W_2(\theta_2^1), W_2(\theta_2^2), W_3(\theta_3^1), W_3(\theta_3^2)) \\
&= \left(\frac{41}{0.7}, \frac{29}{0.3}, \frac{45}{0.7}, \frac{25}{0.3}, \frac{39}{0.5}, \frac{31}{0.5}\right) \\
&\approx (58.5, 96.6, 64.2, 83.3, 78, 62).
\end{aligned}
$$

The diversity among $W_i(\theta_i)$ was smaller than Example 3.

Example 4: (Near complete information) Even if information structure is nearly complete information, the convergence result fails to hold. Consider the same situation as in Example 3 except the joint probability distribution of types.

	θ_2^1	θ_2^2
θ_1^1	$1-3\varepsilon$	ε
θ_1^2	0	0

θ_3^1

	θ_2^1	θ_2^2
θ_1^1	ε	0
θ_1^2	ε	0

θ_3^2

Figure 2: Near complete information case

If ε is close to zero, the information structure is near to complete information $\Theta = \{(\theta_1^1, \theta_2^1, \theta_3^1)\}$. However, as $\rho \to 1$ and $\varepsilon \to 0$, we obtain that $v_1^*(\theta_1^1) = 50$, $v_1^*(\theta_1^2) = -10$, $v_2^*(\theta_2^1) = 50$, $v_2^*(\theta_2^2) = 10$, $v_3^*(\theta_3^1) = 50$, $v_3^*(\theta_3^2) = 30$.

7 Concluding Remark

We examined a noncooperative bargaining game with incomplete information and specified the NBS which has a noncooperative foundation. We showed that the convergence result of the SPBE of the noncooperative bargaining game to the ANBS does not necessarily hold under incomplete information.

From the results in this paper, we would have some comments on the Nash program under incomplete information. The clear relationship between the equilibrium outcome of a noncooperative bargaining game and the NBS exists under complete information. Even if the bargaining procedure is unchanged, the relationship is weakened under incomplete information. In this paper, we showed a necessary and sufficient condition for the limit SSPE of the noncooperative bargaining game to belong to the set of the ANBS under incomplete information. But, the ANBS under incomplete information is not appropriate for the interim solution concept of the Bayesian bargaining game because the interim payoffs for types of each player are indetermined by the maximization problem of the NBS. Furthermore, the SPBE might fail to exist for large ρ under incomplete information. In this case, the convergence result of the SPBEs to the ANBS does not hold. As a result, our attempt to the Nash program under incomplete information is incomplete, and many problems are open to the question.

First, we considered only the bargaining game satisfying both CM condition and I condition on the joint prior probability of types. By the limitations, our arguments can not apply to the 2-person bargaining problem straightforwardly. We should relax these conditions. Second, we considered only the static mechanisms for the proposer in the bargaining game, but the dynamic mechanism design should be allowed as in Mezzetti (2004). Dynamic mechanisms might expand the feasible set of payoff allocations even in more general settings.

Appendix

A. Proof of Theorem 2.

Kosenok and Severinov (2008) have established the following surprising result as a Corollary of their main Theorem (Theorem 1):

Corollary 1. (Kosenok and Severinov) *Consider any ex-ante socially rational decision rule $d(\theta)$, and suppose that the prior p is I and CM condition holds for all agents. Then for any collection of nonnegative constants $v_j(\theta_j)$ satisfying:*

$$\sum_{i\in N}\sum_{\theta_i\in\Theta_i} v_i(\theta_i)p_i(\theta_i) = \sum_{i\in N}\sum_{\theta\in\Theta} u_i(d(\theta),\theta)p(\theta), \tag{15}$$

there exists an BIC, BB, and IR Bayesian mechanism $(d(\theta), t(\theta))$ such that the expected surplus of type θ_i of agent i in this mechanism is equal to $v_i(\theta_i)$.

We check that the expected payoff vector $((R_i^i(\theta_i))_{\theta_i\in\Theta_i}, (V_j^i(\theta_j))_{\theta_j\in\Theta_j})_{j\in N, j\neq i})$ in Theorem 2 satisfies the above condition (15). We have

$$\begin{aligned}
&\sum_{\theta_i\in\Theta_i} R_i^i(\theta_i)p_i(\theta_i) + \sum_{j\in N, j\neq i}\sum_{\theta_j\in\Theta_j} V_j^i(\theta_j)p_j(\theta_j) \\
&= \sum_{\theta_i\in\Theta_i}\left[W_i(\theta_i) - \sum_{\theta_{-i}\in\Theta_{-i}} p_i(\theta_{-i}|\theta_i)\sum_{j\in N, j\neq i}\rho v_j(\theta_j)\right]p_i(\theta_i) + \sum_{j\in N, j\neq i}\sum_{\theta_j\in\Theta_j}\rho v_j(\theta_j)p_j(\theta_j) \\
&= \sum_{\theta_i\in\Theta_i}\sum_{\theta_{-i}\in\Theta_{-i}}\left[\max_{d\in D}\sum_{j\in N} u_i(d,(\theta_{-i},\theta_i))p(\theta_{-i},\theta_i)\right] \\
&\qquad - \sum_{\theta_i\in\Theta_i}\sum_{\theta_{-i}\in\Theta_{-i}} p(\theta_{-i},\theta_i)\sum_{j\in N, j\neq i}\rho v_j(\theta_j) + \sum_{j\in N, j\neq i}\rho v_j(\theta_j)p_j(\theta_j) \\
&= \sum_{\theta\in\Theta}\left[\max_{d\in D}\sum_{j\in N} u_j(d,(\theta))p(\theta)\right] = \sum_{i\in N}\sum_{\theta\in\Theta} u_i(d^*(\theta),(\theta))p(\theta),
\end{aligned}$$

where $d^*(\theta) \in \arg\max_{d \in D} \sum_{j \in N} u_j(d, \theta)$. Thus, the payoff vector satisfies condition (15). Therefore, it leads to Theorem 2.

B. Proof of Theorem 3.

Suppose that mechanism μ^0 is implemented with probability ρ and the disagreement point d^* is realized with probability $1 - \rho$ when player j rejects any proposal. Let us denote $v_j(\theta_j)$ the expected payoff for type θ_j of player j when μ^0 is implemented. Because a payoff for player j with type θ_j is assumed to be zero if d^* is realized, player j obtains the expected payoff $\rho v_j(\theta_j)$. In addition, let us define the expected residual surplus for type θ_i of player i by

$$R_i^i(\theta_i) = W_i(\theta_i) - \sum_{\theta_{-i} \in \Theta_{-i}} p(\theta_{-i}|\theta_i) \sum_{j \in N, j \neq i} \rho v_j(\theta_j).$$

We have the following lemma:

Lemma 2. *Assume that $R_i^i(\theta_i) \geq \rho v_i(\theta_i)$ and $\rho v_j(\theta_j) \geq 0$ for all $j \in N$ and all $\theta_j \in \Theta_j$. There exists an PBE of the informed principal game with principal i in which player i proposes mechanism μ^{i*}, the proposal is accepted by all players, and μ^{i*} is implemented.*

Proof. In the PBE, all types of player i offer mechanism μ^* by inscrutability principle. Then, the beliefs about player i's type by the responders $j \in N$, $j \neq i$ are unchanged. μ^{i*} gives the responders of player $j \in N$, $j \neq i$, with type θ_j the expected payoff of $\rho v_j(\theta_j)$. Every responder is indifferent between the acceptance and the rejection of the proposal because the same payoff is obtained. Thus, the acceptance of the proposal μ^{i*} is (locally) optimal for every type of any player $j \in N$, $j \neq i$. Furthermore, these acceptances transmit no information about players' types. By Theorem 2, mechanism μ^{i*} is BIC under the initial beliefs $p_j(\theta_{-j}|\theta_j)$ for all $j \in N$. Then, it is locally optimal for all players to report their true type under mechanism μ^{i*}. Therefore, it is sufficient to show that any type of the principal i has no incentive to deviate from proposing μ^{i*} to other mechanism μ. Type θ_i of player i obtains the expected payoff of $R_i^i(\theta_i)$ by proposing μ^{i*}, and he obtains $\rho v_i(\theta_i)$ if his proposal is rejected. Since $R_i^i(\theta_i) \geq \rho v_i(\theta_i)$, it is not optimal for player i with type θ_i to make an unacceptable proposal.

Fix an arbitrary mechanism $\mu = (S_1, \ldots, S_n; x^\mu(\cdot), t^\mu(\cdot))$, where S_j is a message space for player $j \in N$. Consider a finite game $G^i(\mu)$ as follows. Player i is a proposer to design a mechanism. In the first stage of $G^i(\mu)$, the proposer i has two choices; exit and proposing μ. If she chooses the exit, type θ_i gets $R_i^i(\theta_i)$ immediately. Otherwise, she offers mechanism μ. Let $U_i(\mu^{i*}|\theta_i)$ be the payoff when she chooses the exit option. In the next stage, all other players accept or reject the proposal μ. In the last stage, the mechanism μ is implemented if all players accept it. If some player rejects the proposal, each type θ_j including player i gets the payoff of $\rho v_j(\theta_j)$.

Since the game $G^i(\mu)$ has only finite periods, there exists an PBE (τ, γ, β) of $G^i(\mu)$, using the existence theorem of Nash equilibrium[3]. Let (τ, γ, β) denotes the probability $\tau_i(\mu|\theta_i)$ with which type θ_i of the proposer i offers mechanism μ, the probability $\tau_j(\mu|\theta_j)$ with which type θ_j accept μ, the probability measure $\gamma_i(\cdot|\theta_i, \mu)$ on S_i^μ representing the message strategy for type θ_i under mechanism μ, the belief $\beta_j^R(\cdot|\theta_j, \mu)$ of type θ_j about other types θ_{-j} when μ is offered by player i, and the belief $\beta_j^I(\cdot|\theta_j, \mu)$ about other types θ_{-j} when the mechanism is implemented.

Let $s = (s_1, \ldots, s_n)$ be the profile of messages in implementation of the

[3]For the existence of sequential equilibrium by Kreps and Wilson (1982), we need to assume the finiteness of the set of feasible mechanisms. However, because we apply the weaker solution concept of perfect Bayesian equilibrium, we do not need an additional assumption. See, also, Fudenberg and Tirole (1991).

mechanism. We will show that the probability with which proposer i offers μ must be zero; $\tau_i(\mu|\theta_i) = 0$ in the PBE. The expected payoff for type θ_i of player i conditional on μ and (τ, γ, β) is given by

$$U_i(\tau, \gamma, \beta|\mu, \theta_i)$$
$$= \sum_{\theta_{-i} \in \Theta_{-i}} \beta_i^R(\theta_i|\theta_i, \mu) \left[\sum_{s \in S} (u_i(x^\mu(s), \theta) + t_i^\mu(s)) \prod_{j \in N} \gamma_j(s_j|\theta_j, \mu) \right] \prod_{j \in N \setminus i} \tau_j(\mu|\theta_j).$$

From the subgame perfection of player i's proposal, it follows that

$$\tau_i(\mu|\theta_i) = \begin{cases} 1 & \text{if} \quad U_i(\tau, \gamma, \beta|\mu, \theta_i) > U_i(\mu^{i*}|\theta_i), \\ 0 & \text{if} \quad U_i(\tau, \gamma, \beta|\mu, \theta_i) < U_i(\mu^{i*}|\theta_i). \end{cases} \tag{16}$$

First, let us show that $U_i(\mu^{i*}|\theta_i) \geq U_i(\tau, \gamma, \beta|\mu, \theta_i)$ for all $\theta_i \in \Theta_i$. The proof is given by contradiction. Suppose that there exists $\hat{\theta}_i \in \Theta_i$ such that $U_i(\hat{\theta}_i|\mu, \tau, \gamma, \beta) > U_i(\mu^{i*}|\theta_i)$. The subgame perfection implies $\tau_i(\mu|\hat{\theta}_i) = 1$. By the Bayes rule, the beliefs of type θ_j is

$$\beta_j^R(\theta_{-j}|\theta_j, \mu) = \frac{\tau_i(\mu|\theta_i) p(\theta_{-j}, \theta_j)}{\sum_{\theta_i' \in \Theta_i} \tau_i(\mu|\theta_i') p_{ij}(\theta_i', \theta_j)}, \quad \text{for} \quad j \neq i \in N,$$

where $p_{ij}(\theta_i, \theta_j)$ is the marginal probability distribution of a pair (θ_i, θ_j).

We have the following inequalities:

$$\sum_{j\in N, j\neq i}\sum_{\theta_j\in\theta_j}\rho v_j(\theta_j)\frac{\sum_{\theta'_i\in\Theta_i}\tau_i(\mu|\theta'_i)p_{ij}(\theta'_i,\theta_j)}{\sum_{\theta'\in\Theta_i}\tau_i(\mu|\theta'_i)p_i(\theta'_i)}$$
$$+\sum_{\theta_i\in\Theta_i}U_i(\mu^{i*}|\theta_i)\frac{\tau_i(\mu|\theta_i)p(\theta_i)}{\sum_{\theta'_i\in\Theta_i}\tau_i(\mu|\theta'_i)p_i(\theta'_i)}$$
$$>\sum_{j\in N, j\neq i}\sum_{\theta_j\in\theta_j}U_j(\tau,\gamma,\beta|\mu,\theta_j)\frac{\sum_{\theta'_i\in\Theta_i}\tau_i(\mu|\theta'_i)p_{ij}(\theta'_i,\theta_j)}{\sum_{\theta'_i\in\Theta_i}\tau_i(\mu|\theta'_i)p_i(\theta'_i)}$$
$$+\sum_{\theta_i\Theta_i}U_i(\tau,\gamma,\beta|\mu,\theta_i)\frac{\tau_i(\mu|\theta_i)p_i(\theta_i)}{\sum_{\theta'_i\in\Theta_i}\tau_i(\mu|\theta'_i)p_i(\theta'_i)}$$
$$>\sum_{j\in N, j\neq i}\sum_{\theta_j\in\theta_j}U_j(\tau,\gamma,\beta|\mu,\theta_j)\frac{\sum_{\theta'_i\in\Theta_i}\tau_i(\mu|\theta'_i)p_{ij}(\theta'_i,\theta_j)}{\sum_{\theta'_i\in\Theta_i}\tau_i(\mu|\theta'_i)p_i(\theta'_i)}$$
$$+\sum_{\theta_i\in\Theta_i}U_i(\mu^{i*}|\theta_i)\frac{\tau_i(\mu|\theta_i)p_i(\theta_i)}{\sum_{\theta'_i\in\Theta_i}\tau_i(\mu|\theta'_i)p_i(\theta'_i)}.$$

The first inequality is satisfied because μ^{i*} is an EPE, BB mechanism with the full residual extraction property for each $\theta_i\in\Theta$, and the second inequality is derived from (16). Then, there exists some $\theta_j\in\Theta_j$, $j\neq i$, such that $U_i(\tau,\gamma,\beta|\mu,\theta_j)<\rho v_j(\theta_j)$. Moreover, it should be satisfied that $\tau_j(\mu|\theta_j)>0$. Player j can get $\rho v_j(\theta_j)>0$ by rejecting μ. This implies that it is not (locally) optimal for type θ_j to accept μ. Thus, $\tau_j(\mu|\theta_j)=0$. This is a contradiction. We conclude that $U_i(\tau,\gamma,\beta|\mu,\theta_i)\leq U_i(\mu^{i*}|\theta_i)$ for all $\theta_i\in\Theta_i$.

Then, if $U_i(\tau,\gamma,\beta|\mu,\theta_i)>U_i(\mu^{i*}|\theta_i)$, it implies that $\tau_i(\mu|\theta_i)=0$ for $\theta_i\in\Theta_i$ by subgame perfection. Even if $U_i(\tau,\gamma,\beta|\mu,\theta_i)=U_i(\mu^{i*}|\theta_i)$, we can construct a new PBE with $\tilde{\tau}_i(\mu|\theta_i)=0$. Therefore, we obtain that $\tau_i(\mu|\theta_i)=0$ for any $\theta_i\in\Theta_i$. This means that every type $\theta_i\in\Theta_i$ selects the exit option and gets $U_i(\mu^{i*}|\theta_i)$ with probability one in $G^i(\mu)$. $\square$

Proof of Theorem 3: (Only if) Suppose that there exists an SPBE such that consists of the following strategies and beliefs of the bargaining game $G(\Gamma,w,\rho)$. In every round of the bargaining game, all types of player i offer the mechanism μ^{i*} with probability one. All types of player i accept μ^{j*} which is proposed by other player j. Moreover, they report their types truthfully under mechanism μ^{i*}. The beliefs in stage 3 and 4 of every bargaining round are given by the initial conditional belief $p_j(\theta_{-j}|\theta_j)$ after μ^{i*} is proposed. If player i proposes mechanism $\mu \in \mathfrak{M}$ such that $\mu \neq \mu^{i*}$, each player plays (τ, γ, β) which was considered in $G^i(\mu)$ in Lemma 2. The game $G^i(\mu)$ is "embedded" in the original bargaining game $G(\Gamma, w, \rho)$.

By the stationarity, the same bargaining game begins at the next round even if some player rejects the proposal because beliefs by all players are unchanged and the strategies does not depend on the strategies in the previous round. Therefore, by the rule of the game, the expected payoff vector $((v_i(\theta))_{\theta_i\in\Theta_i})_{i\in N}$ in the SPBE satisfies, for all $i \in N$ and for all $\theta_i \in \Theta_i$,

$$v_i(\theta_i) = w_i \left[W_i(\theta_i) - \sum_{\theta_{-i}\in\Theta_{-i}} p_i(\theta_{-i}|\theta_i) \sum_{j\in N, j\neq i} \rho v_j(\theta_j) \right] + (1-w_i)\rho v_i(\theta_i). \tag{17}$$

If for some θ_i, $v_i(\theta_i) < 0$, then $R_i^i(\theta_i) < \rho v_i(\theta_i)$ or $\rho v_i(\theta_i) < 0$. Thus, it is optimal for type θ_i of player i to offer an unacceptable proposal or to reject the proposal. This contradicts the fact that the above strategy combination is an SPBE. Therefore, (17) has a nonnegative solution.

(If) If (17) has a nonnegative solution; $v_i(\theta_i) \geq 0$ for all θ_i, then $R_i^i(\theta_i) \geq \rho v_i(\theta_i)$ and $\rho v_i(\theta_i) \geq 0$ for all $\theta_i \in \Theta_i$ and $i \in N$. By Lemma 2, there exists a PBE of each informed principal game with principal i. We can apply the PBE to every informed principal game in all bargaining round. $v_i(\theta_i)$ represents the expected payoff for type θ_i of i when mechanism μ^{i*} is implemented with probability w_i. Thus, the PBE satisfies the stationarity assumption.

C. Proof of Theorem 4.

We can prove the theorem in the same way as Theorem 3 in Severinov (2008). Severinov (2008) considered the case in which the expected reservation payoff for the agent is equal to zero; $v_j(\theta_j) = 0$ for all $j \in N$ and $\theta_j \in \Theta_j$. On the other hand, the principal i must give each agent $\rho v_j(\theta_j)$ in our model. It is sufficient to show that the expected payoff vector by any mechanism μ^i except μ^{i*} is blocked by the concept of $B(\Gamma_i)$ satisfying Axioms 1-4.

Let $(U_i(\mu^i|\theta_i))_{\theta_i \in \Theta}$ be the expected payoff vector for each type of player i under BIC mechanism μ^i. Define blocking concept $\hat{B}_i(\hat{\theta}^i)$ as follows:

$$\hat{B}_i(\hat{\mu}^i) = \Bigg\{ y(\cdot) \in \mathbb{R}_+^{|\Theta_i|} \Bigg| \sum_{\theta_i \in \Theta_i} y(\theta_i) p_i(\theta_i) \leq \sum_{\theta \in \Theta} \Big[\max_{d \in D} \sum_{j \in N} u_j(d, \theta) \Big] p(\theta) - \sum_{j \in N \setminus i} \sum_{\theta_j \in \Theta_j} p_j(\theta_j) \rho v_j(\theta_j), \text{ and } y(\hat{\theta}_i) \leq R_i^i(\hat{\theta}_i) \Bigg\}.$$

We can see as in Severinov (2008) that $\hat{B}_i(\hat{\theta}_i)$ satisfies Axioms 1-4 and blocks any payoff vector except $(R_i^i(\theta_i))_{\theta_i \in \Theta_i} = (U_i(\mu^{i*}|\theta_i))_{\theta_i \in \Theta_i}$.

References

[1] Binmore, K.G. (1987), "Perfect Equilibria in Bargaining Models, " in Binmore, K.G. and P. Dasgupta, eds., *The Economics of Bargaining*, Oxford: Blackwell, 77-105.

[2] Binmore, K. G., A. Rubinstein and A. Wolinsky (1986), "The Nash Bargaining Solution in Economic Modelling," *Rand Journal of Economics* **17**, 176-188.

[3] Britz, V., P. J.-J. Herings and A. Predtetchinski (2010), "Noncooperative Support for the Asymmetric Nash Bargaining Solution, " *Journal of Economic Theory* **145**, 1951-1967.

[4] Cremer, J. and R. P. McLean (1988), "Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions," *Econometrica* **56**, 1247-1257.

[5] de Clippel, G. and E. Minelli (2004), "Two-Person Bargaining with Verifiable Information," *Journal of Mathematical Economics* **40**, 799-813.

[6] Fudenberg, D. and J. Tirole (1991), "Perfect Bayesian Equilibrium and Sequential Equilibrium," *Journal of Economic Theory* **53**, 236-260.

[7] Fudenberg, D. and J. Tirole (1991), *Game Theory*, The MIT Press.

[8] Harsanyi, J.C. and R. Selten (1972), "A Generalized Nash Solution for Two-Person Bargaining Games with Incomplete Information," *Management Science* **18**, 80-106.

[9] Hart, S. and A. Mas-Colell (1996), "Bargaining and Value," *Econometrica* **64**, 357-380.

[10] Heifetz, A. and Z. Neeman (2006), "On the Generic (Im)Possibility of Full Surplus Extraction in Mechanism Design," *Econometrica* **74**, 213-233.

[11] Herings, P.J.J. and A. Predtetchinski (2011), "On the Asymptotic Uniqueness of Bargaining Equilibria," *Economics Letters* **111**, 243-246.

[12] Holmstrom, B. and R. Myerson (1983), "Efficient and Durable Decision Rules with Incomplete Information," *Econometrica* **51**, 1799-1820.

[13] Kosenok, G. and S. Severinov (2008), "Individually Rational, Budget-Balanced Mechanisms and Allocation of Surplus," *Journal of Economic Theory* **140**, 126-161.

[14] Kreps, D. and R. Wilson (1982), "Sequential Equilibria," *Econometrica* **50**, 863-894.

[15] Krishna, V. and R. Serrano (1996), "Multilateral Bargaining," *Review of Economic Studies* **63**, 61-80.

[16] Kultti, K. and H. Vartiainen (2010), "Multilateral Non-Cooperative Bargaining in a General Utility Space," *International Journal of Game Theory* **39**, 677-689.

[17] Laruelle, A. and F. Valenciano (2008), "Noncooperative Foundations of Bargaining Power in Committees and Shapley-Shubik Index," *Games and Economic Behaviors* **63**, 341-353.

[18] Maskin, E. and J. Tirole (1992), "The Principal-Agent Relationship with an Informed Principal, II: Common Values," *Econometrica* **60**, 1-42.

[19] Mezzetti, C. (2004), "Mechanism Design with Interdependent Valuations: Efficiency," *Econometrica* **72**, 1617-1626.

[20] McAfee, P. R. and P.J. Reny (1992), "Correlated Information and Mechanism Design," *Econometrica* **60**, 395-421.

[21] Mertens, J.F. and S. Zamir (1985), "Foundation of Bayesian Analysis for Games with Incomplete Information," *International Journal of Game Theory* **10**, 619-632.

[22] Miyakawa, T. (2008), "Noncooperative Foundation of n-Person Asymmetric Nash Bargaining Solution," *Journal of Economics of Kwansei Gakuin University* **62**, 1-18.

[23] Myerson, R. (1979), "Incentive-Compatibility and the Bargaining Problem," *Econometrica* **47**, 61-73.

[24] Myerson, R. (1983), "Mechanism Design by an Informed Principal," *Econometrica* **51**, 1767-1797.

[25] Myerson, R. (1984), "Two-Person Bargaining Problems with Incomplete Information," *Econometrica* **52**, 461-487.

[26] Myerson, R. and M. Satterthwaite (1983), "Efficient Mechanisms for Bilateral Trading," *Journal of Economic Theory* **28**, 265-281.

[27] Nash, J. F. (1950), "The Bargaining Problem," *Econometrica* **18**, 155-162.

[28] Nash, J. F. (1953), "Two-Person Cooperative Games," *Econometrica* **21**, 128-140.

[29] Okada, A. (1996), "A Noncooperative Coalitional Bargaining Game with Random Proposers," *Games and Economic Behavior* **16**, 97-108.

[30] Okada, A. (2007), "International Negotiations on Climate Change: A Noncooperative Game Anaysis of the Kyoto Protocol," in R. Avenhaus and I.W. Zartman, ed. *Diplomacy Games: Formal Models and International Negotiations*, Springer-Verlag.

[31] Okada, A. (2009), "Non-Cooperative Bargaining and the Incomplete Information Core," Discussion Paper, Graduate School of Economics, No. 2009-03.

[32] Rockafellar, R.T. (1970), *Convex Analysis*, Princeton University Press.

[33] Rubinstein, A. (1982), "Perfect Equilibrium in a Bargaining Model," *Econometrica* **50**, 97-109.

[34] Severinov, S. (2008), "An Efficient Solution to the Informed Principal Problem," *Journal of Economic Theory* **141**, 114-133.

[35] Weidner, F. (1992), "The Generalized Nash Bargaining Sluiton and Incentive Compatible Mechanisms," *International Journal of Game Theory* **21**, 109-129.